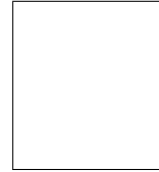




Quiz # 7  
 Math 102-001 Calculus  
 21 July 2016, Thursday  
 Instructor: Ali Sinan Sertöz  
**Solution Key**



Bilkent University

Your Name: .....

Student ID: .....

Your Department: .....

**Q-1)** Find the volume of the solid that lies under the paraboloid  $z = x^2 + y^2$ , above the  $xy$ -plane, and inside the cylinder  $x^2 + y^2 = 2x$ .

*Show your work in detail. Correct answers without justification are never graded.*

This is going to count as an attendance quiz,  
 so everyone who handed in a paper receives full grade.

**Answer:**

*This is Example 4 on page 1025 of the course textbook.*

The solid lies above the disk  $D$  whose boundary circle has equation  $x^2 + y^2 = 2x$  which in polar coordinates is given by

$$D = \{(r, \theta) \mid -\pi/2 \leq \theta \leq \pi/2, 0 \leq r \leq 2 \cos \theta\}.$$

Hence the volume  $V$  can be calculated as follows.

$$\begin{aligned} V &= \iint_D (x^2 + y^2) \, dA = \int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r^3 \, dr \, d\theta \\ &= \int_{-\pi/2}^{\pi/2} \left( \frac{r^4}{4} \Big|_0^{2 \cos \theta} \right) = 4 \int_{-\pi/2}^{\pi/2} \cos^4 \theta \, d\theta = 8 \int_0^{\pi/2} \left( \frac{1 + \cos 2\theta}{2} \right)^2 \, d\theta \\ &= 2 \int_0^{\pi/2} \left[ 1 + 2 \cos 2\theta + \frac{1}{2}(1 + \cos 4\theta) \right] \, d\theta \\ &= 2 \left( \frac{3}{2}\theta + \sin 2\theta + \frac{1}{8} \sin 4\theta \Big|_0^{\pi/2} \right) \\ &= \frac{3\pi}{2}. \end{aligned}$$