Quiz # 7
Math 102-**001** Calculus
21 July 2016, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: .....

Student ID: ..... Your Department: .....

Q-1) Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$.

Show your work in detail. Correct answers without justification are never graded.

This is going to count as an attendance quiz,
so everyone who handed in a paper receives full grade.

Answer:

This is Example 4 on page 1025 of the course textbook.

The solid lies above the disk D whose boundary circle has equation $x^2 + y^2 = 2x$ which in polar coordinates is given by

$$D = \{(r, \theta) \mid -\pi/2 \le \theta \le \pi/2,\ 0 \le r \le 2\cos\theta\ \}.$$

Hence the volume V can be calculated as follows.

$$\begin{aligned}
V &= \iint_D (x^2 + y^2)\, dA = \int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} r^3\, dr d\theta \\
&= \int_{-\pi/2}^{\pi/2} \left(\frac{r^4}{4}\Big|_0^{2\cos\theta} \right) = 4 \int_{-\pi/2}^{\pi/2} \cos^4\theta\, d\theta = 8 \int_0^{\pi/2} \left(\frac{1+\cos 2\theta}{2} \right)^2 d\theta \\
&= 2 \int_0^{\pi/2} \left[1 + 2\cos 2\theta + \frac{1}{2}(1 + \cos 4\theta) \right] d\theta \\
&= 2 \left(\frac{3}{2}\theta + \sin 2\theta + \frac{1}{8}\sin 4\theta \Big|_0^{\pi/2} \right) \\
&= \frac{3\pi}{2}.
\end{aligned}$$