Quiz # 1
Math 102-Section **06** Calculus II
16 February 2017, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: .....

Student ID: ..... Your Department: .....

Show your work in detail. Correct answers without justification are never graded.

Q-1) Write parametric equations for the line of intersection of the two planes

$$x + 2y + 3z = 4 \quad \text{and} \quad 5x + 6y + 7z = 8. \qquad \text{(5 points)}$$

Also write an equation for the plane which is perpendicular to the above line and passes through the point $p = (-1, 1, 1)$. (5 points)

Answer: Call this line L. Since L lies in each of the above planes it is perpendicular to the normals of these planes. Hence L points along

$$\vec{n} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (5\mathbf{i} + 6\mathbf{j} + 7\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 5 & 6 & 7 \end{vmatrix} = -4(\mathbf{i} - 2\mathbf{j} + \mathbf{k}).$$

Check that the above given point $p = (-1, 1, 1)$ lies on both surfaces so lies on L. The line L can be described as a vector in the form

$$L(t) = p + t\vec{n},$$

where $\vec{n} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Now we can write parametric equations for L as follows:

$$\begin{aligned} x &= -1 + t \\ y &= 1 - 2t \\ z &= 1 + t \end{aligned}$$

where $t \in \mathbb{R}$. An equation of the plane perpendicular to this line and passing through $p = (-1, 1, 1)$ is

$$\vec{n} \cdot (x, y, z) = \vec{n} \cdot p.$$

Simplifying we get

$$x - 2y + z = -2.$$