Quiz # 3
Math 102-Section **06** Calculus II
2 March 2017, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: ....

Student ID: .... Your Department: ....

Show your work in detail. Correct answers without justification are never graded.

Q-1) Let $F(x, y, z) = x^2 + y^3z + 5z^{21} - 14$. Assume that x and y are functions of t and s, given as follows.

$$x(t, s) = 1 + 2t + 3s + 3t^3 + 4t^4s^2 + 72s^5t^7,$$
$$y(t, s) = 2 + 4t + 12s + 6t^3 + 5t^4s^4 + 48s^2t^6 + 111s^2t^8.$$

$F(x, y, z) = 0$ defines z as a differentiable function of x and y, and hence as a differentiable function of t and s.

Find $z_t(0, 0)$ at the point $(x, y, z) = (1, 2, 1)$. (10 points)

Answer: Implicitly taking the partial derivative with respect to t of both sides of $F(x, y, z) = 0$, we get

$$2x\, x_t + 3y^2y_tz + y^3z_t + 105z^{20}z_t = 0.$$

Note that $x_t(0, 0) = 2$ and $y_t(0, 0) = 4$, and $(x, y, z) = (1, 2, 1)$. Putting these in we find

$$z_t(0, 0) = -\frac{52}{113}.$$

Another way of solving this is as follows. First you forget about t and s. Differentiating both sides of $F = 0$ with respect to x and y separately and treating z as a function of x and y we get

$$2x + y^3z_x + 105z^{20}z_x = 0,$$
$$3y^2z + y^3z_y + 105z^{20}z_y = 0.$$

From here we obtain at the point $(x, y, z) = (1, 2, 1)$

$$z_x = -\frac{2}{113}, \quad \text{and} \quad z_y = -\frac{12}{113}.$$

Now we use chain rule for z to write

$$z_t = z_xx_t + z_yy_t.$$

Putting in the values of x_t and y_t found above, we find as before

$$z_t(0, 0) = -\frac{52}{113}.$$