



Quiz # 6  
 Math 102-Section 06 Calculus II  
 30 March 2017, Thursday  
 Instructor: Ali Sinan Sertöz  
**Solution Key**



Bilkent University

Your Name: .....

Student ID: .....

Your Department: .....

*Show your work in detail. Correct answers without justification are never graded.*

**Q-1)** Evaluate the integral

$$A = \int_0^{2a} \int_0^{\sqrt{2ay-y^2}} \sqrt{4a^2 - x^2 - y^2} \, dx \, dy,$$

where  $a > 0$ .

**Answer:**

$$\begin{aligned} A &= \int_0^{\pi/2} \int_0^{2a \sin \theta} \sqrt{4a^2 - r^2} \, r \, dr \, d\theta \\ &= -\frac{1}{3} \int_0^{\pi/2} \left( (4a^2 - r^2)^{3/2} \Big|_0^{2a \sin \theta} \right) d\theta \\ &= \frac{8a^3}{3} \int_0^{\pi/2} (1 - \cos^3 \theta) \, d\theta \\ &= \frac{8a^3}{3} \left[ \int_0^{\pi/2} d\theta - \int_0^{\pi/2} (1 - \sin^2 \theta) \cos \theta \, d\theta \right] \\ &= \frac{8a^3}{3} \left[ \frac{\pi}{2} - \int_0^{\pi/2} (1 - \sin^2 \theta) \, d \sin \theta \right] \\ &= \frac{8a^3}{3} \left[ \frac{\pi}{2} - \left( \sin \theta - \frac{1}{3} \sin^3 \theta \Big|_0^{\pi/2} \right) \right] \\ &= \frac{8a^3}{3} \left[ \frac{\pi}{2} - \frac{2}{3} \right] \\ &\approx 2.41 a^3. \end{aligned}$$

