Quiz # 10
Math 102-Section **06** Calculus II
27 April 2017, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Your Name: . . . .

Student ID: . . . . Your Department: . . . .

Show your work in detail. Correct answers without justification are never graded.

Q-1) Consider the series $\sum_{n=2}^{\infty}(-1)^n\frac{\ln n}{n^p}$, where $p>0$ is a constant.

(i) Show that the series converges for all $p>0$. (2 points)

(ii) Show that the series is conditionally convergent for $0<p\leq 1$. (4 points)

(iii) Show that the series is absolutely convergent for $p>1$. (4 points)

Answer: For the following arguments set $a_n=\frac{\ln n}{n^p}$.

(i) Consider the function $f(x)=\frac{\ln x}{x^p}$. Then $f'(x)=\frac{1-p\ln x}{x^{p+1}}<0$ for all large x. Also $f(x)$ goes to zero as x goes to infinity by l'Hospital's rule. Hence the series converges by the Alternating Series Test.

(ii) Here we must show that $\sum_{n=2}^{\infty}a_n$ diverges when $0<p\leq 1$

We have $a_n>\frac{1}{n^p}$, and $\sum_{n=2}^{\infty}\frac{1}{n^p}$ diverges by the p-test. Hence by the Comparison Test our series diverges.

(iii) Here we must show that $\sum_{n=2}^{\infty}a_n$ converges when $p>1$

Let $\epsilon>0$ be such that $p-\epsilon>1$. Then for large n we have $\ln n<n^{\epsilon}$, and hence $a_n<\frac{n^{\epsilon}}{n^p}=\frac{1}{n^{p-\epsilon}}$.
Since $\sum_{n=2}^{\infty}\frac{1}{n^{p-\epsilon}}$ converges by the p-test, our series converges by the Comparison Test.