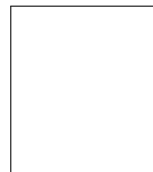




Quiz # 10
Math 102-Section 06 Calculus II
27 April 2017, Thursday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Your Name:

Student ID:

Your Department:

Show your work in detail. Correct answers without justification are never graded.

Q-1) Consider the series $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n^p}$, where $p > 0$ is a constant.

- (i) Show that the series converges for all $p > 0$. (2 points)
- (ii) Show that the series is conditionally convergent for $0 < p \leq 1$. (4 points)
- (iii) Show that the series is absolutely convergent for $p > 1$. (4 points)

Answer: For the following arguments set $a_n = \frac{\ln n}{n^p}$.

(i) Consider the function $f(x) = \frac{\ln x}{x^p}$. Then $f'(x) = \frac{1 - p \ln x}{x^{p+1}} < 0$ for all large x . Also $f(x)$ goes to zero as x goes to infinity by l'Hospital's rule. Hence the series converges by the Alternating Series Test.

(ii) Here we must show that $\sum_{n=2}^{\infty} a_n$ diverges when $0 < p \leq 1$

We have $a_n > \frac{1}{n^p}$, and $\sum_{n=2}^{\infty} \frac{1}{n^p}$ diverges by the p -test. Hence by the Comparison Test our series diverges.

(iii) Here we must show that $\sum_{n=2}^{\infty} a_n$ converges when $p > 1$

Let $\epsilon > 0$ be such that $p - \epsilon > 1$. Then for large n we have $\ln n < n^\epsilon$, and hence $a_n < \frac{n^\epsilon}{n^p} = \frac{1}{n^{p-\epsilon}}$.

Since $\sum_{n=2}^{\infty} \frac{1}{n^{p-\epsilon}}$ converges by the p -test, our series converges by the Comparison Test.