Bilkent University

Quiz # 02
Math 102-Section **10** Calculus II
21 February 2019, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) In the following you may assume without checking that all partial derivatives of all orders exist and are continuous at those points of interest.

(i) Let $f(x,y,z) = x^5 \sin^2(y^3 z^7)$. Calculate f_x and f_y.

(ii) Let $f(x,y,z) = \arctan \dfrac{z^2+y^2}{2\pi} + x^2 y^7 + e^{11x+2y+3z} + 2019$. Calculate $\dfrac{\partial}{\partial x} f_{yy}$.

(iii) Let $f(x,y) = x(x^2+y^3)^{-3/2} e^{\sin xy} + \left(\dfrac{x^3+x}{x^2+y^4+1}\right)^5 \left(\ln(y^4+x^2y^2+1)\right)^7$.

Calculate $f_x(1,0)$.

Grading: (i) 2 points, (ii) 4 points, (iii) 4 points.

Solution:

(i) $f_x = 5x^4 \sin^2(y^3z^7)$, $\qquad f_y = 6x^5y^2z^7 \sin(y^3z^7)\cos(y^3z^7)$.

(ii) First note that due to the continuity assumptions given at the beginning we have

$$\begin{aligned}
\frac{\partial}{\partial x} f_{yy} &= f_{yyx} = f_{xyy} = \frac{\partial}{\partial y}\frac{\partial}{\partial y} f_x \\
&= \frac{\partial}{\partial y}\frac{\partial}{\partial y}(2xy^7 + 11e^{11x+2y+3z}) \\
&= \frac{\partial}{\partial y}(14xy^6 + 22e^{11x+2y+3z}) \\
&= 84xy^5 + 44e^{11x+2y+3z}.
\end{aligned}$$

(iii) Let $g(x) = f(x,0) = \dfrac{1}{x^2}$. Then

$$f_x(1,0) = g'(1) = \left(-\frac{2}{x^3}\bigg|_{x=1} \right) = -2.$$