Bilkent University

Quiz # 11
Math 102-Section **10** Calculus II
2 May 2019, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Let $s_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$.

(i) Show that there exists an integer n such that $2019 < s_n < 2020$.

(ii) Explicitly find an integer n such that $10 < s_n < 11$.

Grading: (i) 5 points, (ii) 5 points. Hint: $e^{10} = 22026.46...$ and $\ln(n+1) < s_n < 1 + \ln n$.

Solution:

(i) Since the harmonic series diverges to infinity, the partial sum s_n eventually surpasses every real number.

In particular there exists an integer n such that $s_{n-1} \leq 2019 < s_n$. Note that in this case $n > 2$ since $s_2 = 3/2 < 2$.

We need to show that for this n we must have $s_n < 2020$. But $s_n - s_{n-1} = 1/n < 1/2$ since $n > 2$.

Hence $s_n - 2019 < 1/2$, which in turn shows that $s_n < 2020$ as claimed.

(ii)

Using the second hint, it suffices to find an n such that

$$10 \leq \ln(n+1) < s_n < 1 + \ln n \leq 11.$$

Using the first hint, the first inequality gives $n \geq 22025.46$, and the last inequality gives $n \leq 22026.46$

Hence $n = 22026$ will work for us. In fact $s_{22026} = 10.57$.

However the smallest n with $10 < s_n < 11$ is 12367. Check that $s_{12366} = 9.999962$ and $s_{12367} = 10.000043$.