Bilkent University

Quiz # 02
Math 102 Section 03 Calculus II
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Solution Key

Q-1) The ellipsoid $16x^2 + 4y^2 + 3z^2 = 160$ intersects the plane $x = 3$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point $(3, 1, 2)$.

Solution:

First check that the point $(3, 1, 2)$ is on the ellipsoid. $16(3^2) + 4(1^2) + 3(2^2) = 160$.

Then putting $x = 3$ and simplifying we get

$$\frac{y^2}{4} + \frac{3z^2}{16} = 1.$$

First Solution:
In the yz-plane situated at $x = 3$, we can consider z as a function of y, and implicitly differentiate the above equation with respect to y to obtain z' on the ellipse. This gives

$$\frac{\partial z}{\partial y} = -\frac{4y}{3z}.$$

Evaluating at the point $(y, z) = (1, 2)$ we get

$$\left.\frac{\partial z}{\partial y}\right|_{(1,2)} = -\frac{2}{3}.$$

Finally, parametric equations for the tangent line to this ellipse at the point $(3, 1, 2)$ are:

$$x(t) = 3, \quad y(t) = 1 + t, \quad z(t) = 2 - \frac{2}{3}t, \quad t \in \mathbb{R}.$$

Second Solution:
We parametrize the above ellipse as

$$y(\theta) = 2\cos\theta, \quad z(\theta) = \frac{4}{\sqrt{3}}\sin\theta.$$

The point $(y, z) = (1, 2)$ corresponds to $\theta = \pi/3$. We have

$$y'(\theta) = -2\sin\theta,\ z'(\theta) = \frac{4}{\sqrt{3}}\cos\theta, \quad \text{and} \quad y'(\pi/3) = -\sqrt{3},\ z'(\pi/3) = \frac{2}{\sqrt{3}}.$$

Hence parametric equations for the tangent line at $(3, 1, 2)$ are

$$x(s) = 3, \quad y(s) = 1 - \sqrt{3}s, \quad z(s) = 2 + \frac{2}{\sqrt{3}}s, \quad s \in \mathbb{R}.$$

Note that $s = -t/\sqrt{3}$ gives the previous parametrization.