Bilkent University

Quiz # 03
Math 102 Section 03 Calculus II
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Solution Key

Q-1) Suppose that $F(x,y,z) = x^3 + y^2 - z^4 - 236 = 0$ defines z as a differentiable function of x and y. Further assume that x and y are functions of the variables s and t as follows.

$$x(s,t) = 2 + t^3 + st^2 + s^4$$
$$y(s,t) = 3 + 5t + 12s + 3t^3 + 7t^3s^4 + 50s^2t + 2020s^4t^5$$

Taking $z > 0$, calculate $\left.\dfrac{\partial z}{\partial t}\right|_{(s,t)=(1,0)}$.

Solution:

We note that $x(1,0) = 3$, and $y(1,0) = 15$.

From $F(3,15,z) = 0$ we get $z(1,0) = 2$, since we are interested in the case when $z > 0$.

Since x, y and hence z are functions of s and t, we take the partial derivative with respect to t of both sides of the equation $F(x,y,z) = 0$ to get

$$3x^2x_t + 2yy_t - 4z^3z_t = 0. \qquad (*)$$

We also find directly that

$$x_t = 3t^2 + 2st, \quad y_t = 5 + 9t^2 + 21t^2s^3 + 50s^2 + 10100s^4t^4$$

and

$$x_t(1,0) = 0, \quad y_t(1,0) = 55.$$

Putting these values in $(*)$, we get

$$z_t(1,0) = \frac{825}{16}.$$