Bilkent University

Quiz # 05
Math 102 Section 03 Calculus II
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Solution Key

Q-1) Let $f(x, y) = x^3 + xy^2 + y^2 - 3x$. Classify the critical points of f.

Hint: $f(-1+s, 0) - f(-1, 0) = s^2(s-3)$ and $f(-1+\frac{t^2}{3}, t) - f(-1, 0) = \frac{t^6}{26}$.

Solution:

Solving $f_x = 3x^2 + y^2 - 3 = 0$ and $f_y = 2xy + 2y = 0$ simultaneously, we get $(1, 0)$ and $(-1, 0)$ as the only critical points.

To classify these critical points we use the second derivative test.

$f_{xx} = 6x$, $f_{xy} = 2y$, $f_{yy} = 2x + 2$, and $\Delta = f_{xx}f_{yy} - f_{xy}^2 = 12x^2 + 12x - 4y^2$.

Since $\Delta(1, 0) = 24 > 0$, and $f_{xx}(0, 1) = 6 > 0$, the critical point $(1, 0)$ **is a local minimum point.**

Since $\Delta(-1, 0) = 0$, the test fails at this point. Hence we must examine the behaviour of f around the point $(-1, 0)$.

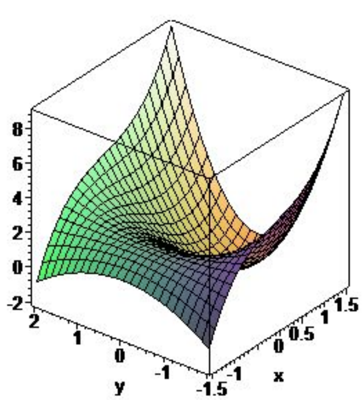
Using the first hint, we see that $f(-1+s, 0) - f(-1, 0) = s^2(s-3) \leq 0$ for all s with $-3 < s < 3$.

This shows that in every neighborhood of $(-1, 0)$, there are points (x, y), namely $(-1+s, 0)$, such that $f(x, y) \leq f(-1, 0)$.

Using the second hint we see that $f(-1+\frac{t^2}{3}, t) - f(-1, 0) = \frac{t^6}{26} \geq 0$.

This shows that in every neighborhood of $(-1, 0)$, there are points (x, y), namely $(-1+t^2/3, t)$, such that $f(x, y) \geq f(-1, 0)$.

These two properties show that the critical point $(-1, 0)$ **is a saddle point.**



The graph shows how the surface $z = f(x, y)$ looks like around $(-1, 0)$.