Bilkent University

Quiz # 07
Math 102 Section 03 Calculus II
9 April 2020
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Solution Key

Q-1) Let D be the region in the xy-plane bounded by the circles $x^2+y^2=2x$ and $x^2+y^2=2y$. Let R be the solid lying above D and below the sphere $x^2+y^2+z^2=4$.

(i) Set up an integral in Cartesian coordinates which calculates the volume of R.

(ii) Set up an integral in cylindrical coordinates which calculates the volume of R.

(iii) Go to www.wolframalpha.com, or use any other computer algebra sytem, and evaluate both of the above integrals. Make sure that they evaluate to the same value. Then write down the volume of R.

Grading: (i) 4 points, (ii) 4 points, (iii) 2 points

Solution:

(i) The integral in Cartesian coordinates is

$$Vol(R)=\int_{y=0}^{y=1}\int_{x=1-\sqrt{1-y^2}}^{x=\sqrt{2y-y^2}}\sqrt{4-x^2-y^2}\,dx\,dy.$$

(ii) The integral in cylindrical coordinates is

$$Vol(R)=2\int_{\theta=0}^{\theta=\pi/4}\int_{r=0}^{r=2\sin\theta}\sqrt{4-r^2}\,r\;dr\,d\theta.$$

Here we multiply the integral by 2 due to symmetry. If symmetry is not observed, we could write

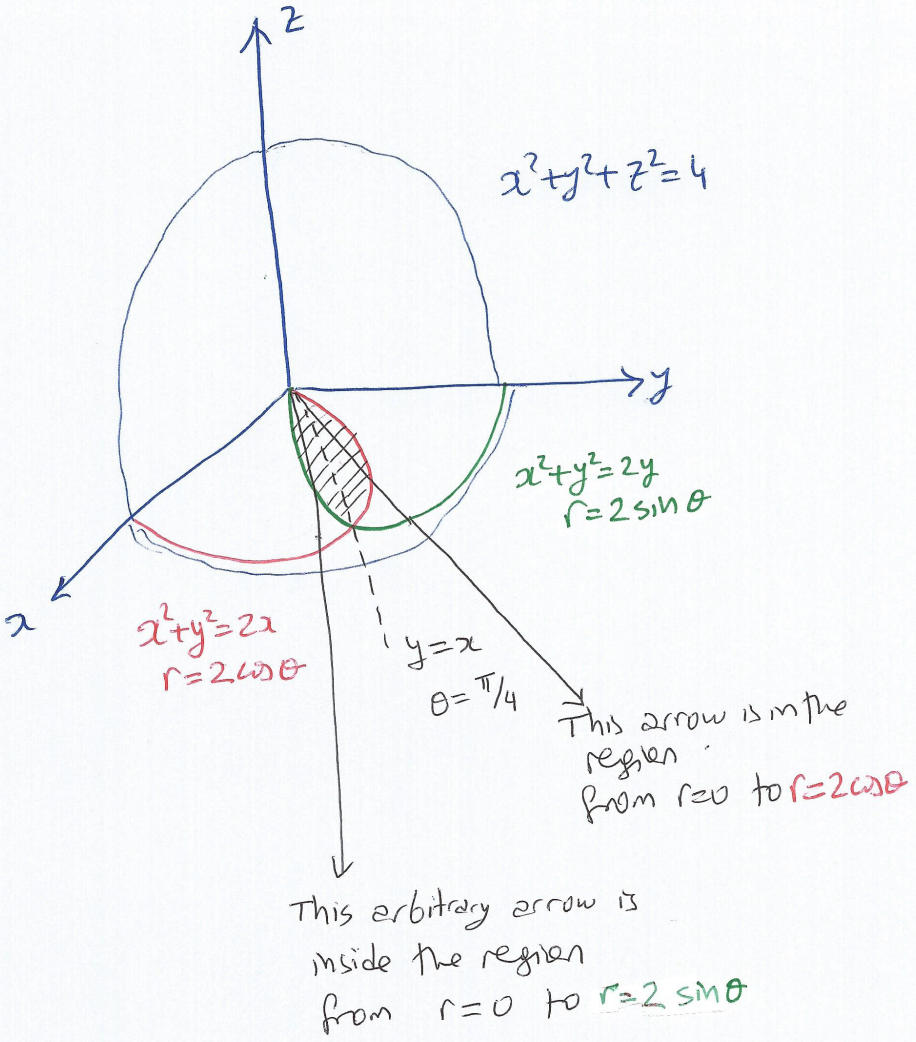
$$Vol(R)=\int_{\theta=0}^{\theta=\pi/4}\int_{r=0}^{r=2\sin\theta}\sqrt{4-r^2}\,r\;dr\,d\theta+\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=2\cos\theta}\sqrt{4-r^2}\,r\;dr\,d\theta.$$

Observe here that both integrals evaluate to the same value.

(iii) You will probably get only a floating value for the value of the Cartesian integral since it is too complicated. But for the cylindrical integral you may get an exact answer.

$$Vol(R)=\frac{4}{3}\pi-\frac{20}{9}\sqrt{2}\approx 1.046093400$$

See next page for a figure.



D is the shaded region lying between the two circles.