Bilkent University

Quiz # 05
Math 102 - Calculus II - Section 03
17 March 2022 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) We are given two planes $P1$ and $P2$ by the equations

$$\begin{aligned} P1: &\quad 3x+5y+7z=34 \\ P2: &\quad 4x+6y+z=40. \end{aligned}$$

(a) Find a point which lies on both of these planes.

(b) Write parametric equations for the line of intersection of these two planes.

(c) Write an equation of the form $Ax+By+Cz=D$ for the plane which passes through the point $(1,2,6)$ and is perpendicular to the line of intersection of the planes $P1$ and $P2$.

Grading: 2+4+4 points

Solutions:

(a) This you do by trial and error. One such point is $q=(-2,8,0)$.

(b) The vectors $N1=(3,5,7)$ and $N2=(4,6,1)$ are orthogonal to the planes $P1$ and $P2$ respectively. The vector

$$V=N1\times N2=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 7 \\ 4 & 6 & 1 \end{vmatrix}=(-37,25,-2)$$

is parallel to these two planes and is in the direction of the line of intersection. Hence parametric equations for this line is

$$x=-2-37t, \quad y=8+25t, \quad z=-2t, \quad t\in\mathbb{R}.$$

(c) The above vector V is orthogonal to this plane hence and equation will be

$$V\cdot(x,y,z)=V\cdot(1,2,6),$$

which simplifies to

$$-37x+25y-2z=1.$$

An alternate way to obtain V: Solve the system

$$\begin{aligned} 3x+5y+7z&=34 \\ 4x+6y+z&=40. \end{aligned}$$

for x and y in terms of z to obtain

$$x=-2+\frac{37}{2}z,\ y=8-\frac{25}{2}z.$$

Putting $z = 0$ and $z = 2$ gives us two points along the line L of intersection.

$$p = (-2, 8, 0) \text{ and } q = (35, -17, 2).$$

Then $V = p - q = (-37, 25, -2)$.