Bilkent University

Quiz # 07
Math 102 - Calculus II - Section 03
31 March 2022 Thursday
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Solution Key

Q-1) Let S be the surface defined by $z = f(x, y)$ for some function f. We know that the line $\vec{r}(t) = (1+4t, 2+5t, 3+6t)$, $t \in \mathbb{R}$, is tangent to S when $t = 0$. We also know that the plane $x+2y-z = 2$ intersects S orthogonally at $(1, 2, 3)$.

(a) Find $f(1, 2)$.

(b) Write an equation for the tangent plane of S at $(1, 2, f(1, 2))$ in the form $Ax + By + Cz = D$ where $A \geq 0$.

(c) Using a linear approximation estimate $f(\frac{16}{17}, \frac{17}{10})$.

Grading: 1+4+5 points

Solutions:

(a) Since $\vec{r}(t)$ is tangent to the surface at $t = 0$, $\vec{r}(0) = (1, 2, 3)$ is on the surface. Hence $f(1, 2) = 3$.

(b) The vector $\vec{r}(t)' = (4, 5, 6)$ is tangent to S at $p_0 = (1, 2, 3)$ and hence $(4, 5, 6)$ is orthogonal to the normal vector $\vec{n}$ of S. Also the normal vector $\vec{u} = (1, 2, -1)$ of the plane $x + 2y - z = 2$ is also orthogonal to $\vec{n}$. Thus

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & 6 \\ 1 & 2 & -1 \end{vmatrix} = (-17, 10, 3).$$

Then the equation of the tangent plane to S at $p_0 = (1, 2, 3)$ is

$$-17(x - 1) + 10(y - 2) + 3(z - 3) = 0,$$

or equivalently

$$17x - 10y - 3z = -12.$$

(c) From the above tangent equation we solve for z to find the linearization

$$L(x, y) = z = 4 + \frac{17}{3}x - \frac{10}{3}y.$$

Then

$$f(\tfrac{16}{17}, \tfrac{17}{10}) \approx L(\tfrac{16}{17}, \tfrac{17}{10}) = \frac{5}{3}.$$