



Bilkent University

Quiz # 08
Math 102 - Calculus II - Section 03
7 April 2022 Thursday
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Solution Key

Q-1) We have the following functions:

$$f(x, y) = x^3 + 7x^2y - y^4 + 30y + 1$$

$$g(t, s) = 27t + 2st + s^4 - 2$$

$$h(t, s) = t^7 - s^6 + 3$$

$$P(t, s) = f(g(t, s), h(t, s)).$$

(a) Calculate $\frac{\partial P}{\partial t}(0, 1)$.

(b) Calculate $\frac{\partial P}{\partial s}(0, 1)$.

Grading: 5+5 points

Solutions:

First Method: Here we simply calculate P and its partial derivatives.

$$P(t, s) = (27t + 2ts + s^4 - 2)^3 + 7(27t + 2ts + s^4 - 2)^2(t^7 - s^6 + 3) - (t^7 - s^6 + 3)^4 + 30t^7 - 30s^6 + 91$$

$$\frac{\partial P}{\partial t}(t, s) = 3(27t + 2ts + s^4 - 2)^2(27 + 2s) + 14(27t + 2ts + s^4 - 2)(t^7 - s^6 + 3)(27 + 2s) + 49(27t + 2ts + s^4 - 2)^2t^6 - 28(t^7 - s^6 + 3)^3t^6 + 210t^6$$

$$\frac{\partial P}{\partial s}(t, s) = 3(27t + 2ts + s^4 - 2)^2(2t + 4s^3) + 14(27t + 2ts + s^4 - 2)(t^7 - s^6 + 3)(2t + 4s^3) - 42(27t + 2ts + s^4 - 2)^2s^5 + 24(t^7 - s^6 + 3)^3s^5 - 180s^5$$

Then we have

$$\frac{\partial P}{\partial t}(0, 1) = -725, \quad \frac{\partial P}{\partial s}(0, 1) = -130.$$

Second Method: Here we use the chain rule and hence calculate the following:

$$\begin{aligned} g(0, 1) &= -1, & h(0, 1) &= 2 \\ f_x(x, y) &= 3x^2 + 14xy, & f_y(x, y) &= 7x^2 - 4y^3 + 30 \\ g_t(t, s) &= 27 + 2s, & g_s(t, s) &= 2t + 4s^3 \\ h_t(t, s) &= 7t^6, & h_s(t, s) &= -6s^5 \end{aligned}$$

Next we evaluate these at the relevant points.

$$f_x(-1, 2) = -25,$$

$$g_t(0, 1) = 29,$$

$$h_t(0, 1) = 0,$$

$$f_y(-1, 2) = 5$$

$$g_s(0, 1) = 4$$

$$h_s(0, 1) = -6$$

Finally we use chain rule:

$$P_t(0, 1) = f_x(-1, 2)g_t(0, 1) + f_y(-1, 2)h_t(0, 1) = (-25)(29) + (5)(0) = -725,$$

$$P_s(0, 1) = f_x(-1, 2)g_s(0, 1) + f_y(-1, 2)h_s(0, 1) = (-25)(4) + (5)(-6) = -130.$$