Bilkent University

Quiz # 6
Math 102-Section 09
3 May 2023, Wednesday, Moodle Quiz
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Solution Key

Q-1) Consider the function $f(x,y,z) = 5x^2 + 3y^2 - z^3 + 10$.

(a) Write the gradient $\nabla f(x,y,z)$.

(b) Write $\nabla f(1,2,3)$.

(c) Write the equation of the tangent plane to the surface $f(x,y,z) = 0$ at the point $(1,2,3)$ in the form $z = A + Bx + Cy$, where A, B, C are real numbers.

(d) Find the x, y and z intercepts of this tangent plane.

(e) Calculate the volume of the pyramid bounded by this tangent plane and the $x = 0$, $y = 0$, $z = 0$ planes. Write your answer in the form $\frac{D}{E}$ where D and E are positive integers with no common factor. (Hint: Volume of a pyramid is $\frac{1}{3} \times$ base area $\times$ height.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 1+1+3+3+2=10 points.

Solution:

(1-a) $\nabla f(x,y,z) = (10x, 6y, -3z^2)$.

(1-b) $\nabla f(1,2,3) = (10, 12, -27)$.

(1-c) An equation of this tangent plane is of the form

$$\nabla f(1,2,3) \cdot (x,y,z) = \nabla f(1,2,3) \cdot (1,2,3), \quad \text{or after expanding } \; 10x + 12y - 27z = -47,$$

which when expressed as in the problem takes the form

$$z = \frac{47}{27} + \frac{10}{27}x + \frac{12}{27}y.$$

(1-d) We use the equation $10x + 12y - 27z = -47$. Putting in $y = z = 0$ gives the x-intercept. Similarly for the y and z intercepts. These then turn out to be

$$x_0 = -\frac{47}{10}, \; y_0 = -\frac{47}{12}, \; z_0 = \frac{47}{27}.$$

(1-e) The required volume is

$$V = \frac{1}{3} \times \text{base area} \times \text{ height} = \frac{1}{3} \times (\frac{1}{2}|x_0|\,|y_0|) \times (|z_0|) = \frac{103823}{19440}.$$