Quiz # 01
Math 102 Section 10 Calculus II
2 February 2026, Monday
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Solution Key

Bilkent University

We define a sequence (a_n) as follows: $a_0 = 1$, $a_{n+1} = \dfrac{3a_n}{1+a_n}$, for $n \geq 0$.

(i) Show that if $a_n < 2$ for some $n \geq 0$, then $a_{n+1} < 2$.

(ii) Show that (a_n) is an increasing sequence.

(iii) Show that this sequence converges.

(iv) Find $\lim_{n\to\infty} a_n$.

Grading: 2+2+3+3=10 points

Grader: xx

Solution:

(i) Assuming $a_n < 2$, we add $2a_n$ to both sides of this inequality to obtain $3a_n < 2+2a_n = 2(1+a_n)$, which then gives $a_{n+1} = \dfrac{3a_n}{1+a_n} < 2$ as claimed.

(ii) Since $a_n < 2$ for all $n \geq 0$ as shown above, we must have $1+a_n < 3$, or equivalently $\dfrac{1}{1+a_n} > \dfrac{1}{3}$. Multiplying both sides by $3a_n$ we get $a_{n+1} = \dfrac{3a_n}{1+a_n} > \dfrac{3a_n}{3} = a_n$, giving us $a_{n+1} > a_n$ as required.

(iii) We just showed that this sequence is increasing and bounded from above by 2. By the bounded monotone sequence theorem this sequence converges.

(iv) Let $\lim_{n\to\infty} a_n = L$. Taking the limit of both sides of the recursion formula we get $L = \dfrac{3L}{1+L}$, which gives $L = 0$ or $L = 2$. Since the sequence is positive and increasing, the limit cannot be zero. So $L = 2$.