Quiz # 01
Math 102 Section 11 Calculus II
3 February 2026, Tuesday
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Solution Key

Bilkent University

We define a sequence (a_n) as follows: $a_0 = 8$, $a_{n+1} = \dfrac{7a_n}{1+a_n}$, for $n \geq 0$.

(i) Show that if $a_n > 6$ for some $n \geq 0$, then $a_{n+1} > 6$.

(ii) Show that (a_n) is a decreasing sequence.

(iii) Show that this sequence converges.

(iv) Find $\lim\limits_{n\to\infty} a_n$.

Grading: 2+2+3+3=10 points

Grader: TBA

Solution:

(i) Assuming $a_n > 6$, we add $6a_n$ to both sides of this inequality to obtain $7a_n > 6+6a_n = 6(1+a_n)$, which then gives $a_{n+1} = \dfrac{7a_n}{1+a_n} > 6$ as claimed.

(ii) Since $a_n > 6$ for all $n \geq 0$ as shown above, we must have $1+a_n > 7$, or equivalently $\dfrac{1}{1+a_n} < \dfrac{1}{7}$. Multiplying both sides by $7a_n$ we get $a_{n+1} = \dfrac{7a_n}{1+a_n} < \dfrac{7a_n}{7} = a_n$, giving us $a_{n+1} < a_n$ as required.

(iii) We just showed that this sequence is decreasing and bounded from below by 6. By the bounded monotone sequence theorem this sequence converges.

(iv) Let $\lim\limits_{n\to\infty} a_n = L$. Taking the limit of both sides of the recursion formula we get $L = \dfrac{7L}{1+L}$, which gives $L = 0$ or $L = 6$. Since the sequence is bounded from below by 6, the limit cannot be zero. So $L = 6$.