Quiz # 02
Math 102 Section 10 Calculus II
9 February 2026, Monday
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Solution Key

Bilkent University

Check the following series for convergence.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n^2}}}$.

(ii) $\displaystyle\sum_{n=1}^{\infty} a_n$, where $a_n = \dfrac{5n^3+n^2+n-2}{4n^4+3n^3-n}$.

(iii) $\displaystyle\sum_{n=1}^{\infty} b_n$, where $b_n = \dfrac{4n^3+n^2+n-2}{\sqrt{n}(5n^4+3n^3-n)}$.

Grading: 3+3+4=10 points

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Solution:

(i) Solution 1: We note that since $\displaystyle\lim_{n\to\infty} n^{\frac{1}{n^2}} = 1$, we must have $n^{\frac{1}{n^2}} < 2$ for all large n. Then $n^{1+\frac{1}{n^2}} = n\ n^{\frac{1}{n^2}} < 2n$ and hence $\dfrac{1}{n^{1+\frac{1}{n^2}}} > \dfrac{1}{2}\dfrac{1}{n}$ for all large n. Since the harmonic series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, our series diverges by the *Direct Comparison Test*.

Note that since the power $n+\frac{1}{n^2}$ is not a fixed number, this series is not a p-series!

Solution 2: Since $\displaystyle\lim_{n\to\infty} \frac{\frac{1}{n^{1+\frac{1}{n^2}}}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{1}{n^{\frac{1}{n^2}}} = 1$, and since the harmonic series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, our series diverges by the *Limit Comparison Test*.

(ii) Since $\displaystyle\lim_{n\to\infty}\frac{a_n}{\frac{1}{n}} = \frac{5}{4}$, and since the harmonic series diverges, our series diverges by the *Limit Comparison Test*.

(iii) Since $\displaystyle\lim_{n\to\infty}\frac{b_n}{\frac{1}{n^{3/2}}} = \frac{4}{5}$, and since $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ converges as a p-series with $p>1$, our series converges by the *Limit Comparison Test*.