Quiz # 03
Math 102 Section 10 Calculus II
16 February 2026, Monday
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Solution Key

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Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n^7 x^n}{17^n}$.

(i) Find its radius of convergence.

(ii) Find its interval of convergence.

Grading: 6+(2+2)=10 points

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Solution:

(i) Let $a_n = \dfrac{n^7 x^n}{17^n}$.

Using the Ratio Test we find: $\displaystyle\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{|x|}{17}$.

Using the Root Test we also find: $\displaystyle\lim_{n\to\infty} |a_n|^{1/n} = \frac{|x|}{17}$.

Thus for absolute convergence we need $\dfrac{|x|}{17} < 1$, or equivalently $|x| < 17$. By both tests, the series diverges if $|x| > 17$.

So the radius of convergence is 17.

(ii) We need to check the end points, $x = 17$ and $x = -17$.

When $x = 17$, the series becomes $\displaystyle\sum_{n\to\infty} n^7$, and when $x = -17$, the series becomes $\displaystyle\sum_{n\to\infty} (-1)^n n^7$. In both cases the series diverges since the general term does not go to zero as n goes to infinity.

Hence the interval of convergence is $(-17, 17)$.