



Quiz # 03
Math 102 Section 11 Calculus II
17 February 2026, Tuesday
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Solution Key

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Consider the power series $\sum_{n=0}^{\infty} \frac{n^2(x-1)^n}{3^n}$.

- (i) Find its radius of convergence.
- (ii) Find its interval of convergence.

Grading: $6+(2+2)=10$ points

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Solution:

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(i) Let $a_n = \frac{n^2(x-1)^n}{3^n}$.

Using the Ratio Test we find: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{|x-1|}{3}$.

Using the Root Test we also find: $\lim_{n \rightarrow \infty} |a_n|^{1/n} = \frac{|x-1|}{3}$.

Thus for absolute convergence we need $\frac{|x-1|}{3} < 1$, or equivalently $|x-1| < 3$. By both tests, the series diverges if $|x-1| > 3$.

So the radius of convergence is 3.

(ii) We need to check the end points, $x = -2$ and $x = 4$.

When $x = 4$, the series becomes $\sum_{n \rightarrow \infty} n^2$, and when $x = -2$, the series becomes $\sum_{n \rightarrow \infty} (-1)^n n^2$. In both cases the series diverges since the general term does not go to zero as n goes to infinity.

Hence the interval of convergence is $(-2, 4)$.