Date: 21 July 2004, Wednesday
Instructor: Ali Sinan Sertöz
Time: 16:40-17:40

NAME:..

STUDENT NO:..

Math 112 Intermediate Calculus II – QUIZ – Solutions

Q-1) Calculate the following limits, if they exist:

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}.$ **b)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^4}.$

Solution:

a) $\displaystyle\left|\frac{x^3+y^3}{x^2+y^2}\right| \leq \frac{|x|x^2+|y|y^2}{x^2+y^2} \leq \frac{|x|(x^2+y^2)+|y|(y^2+x^2)}{x^2+y^2} = |x|+|y|$. Hence the required limit is zero by the sandwich theorem.

a) Approaching the origin along the lines $y=\lambda x$, we see that the limit depends on λ, so no unique limit exists.

Q-2) Find the directional derivative of $f(x,y,z)=2x^3y+yz+xz^2$ at the point $p_0=(1,2,4)$ in the direction of the vector $\vec{v}=(3,4,12)$.

Solution:

$\nabla f(p_0)=(28,6,10)$, $\displaystyle\vec{u}=\frac{\vec{v}}{|\vec{v}|}=(\frac{3}{13},\frac{4}{13},\frac{12}{13})$.

$\displaystyle D_{\vec{u}}f(p_0)=\nabla f(p_0)\cdot\vec{u}=\frac{228}{13}.$

Q-3) Find the critical points of $f(x,y)=x^2y-x^3-xy^2+x+1$ and decide if each critical point is a local min/max or a saddle point. Find global min/max points, if they exist.

Solution:

From $f_y=x^2-2xy=0$ we find that $x=0$ or $x=2y$. Putting $x=0$ into $f_x=2xy-3x^2-y^2+1=0$, we get $y=\pm1$, giving us the critical points $(0,1)$ and $(0,-1)$.

Putting $x=2y$ into $f_x=2xy-3x^2-y^2+1=0$, we get $y=\pm1/3$, giving us the critical points $(2/3,1/3)$ and $(-2/3,-1/3)$.

Applying the second derivative test we find that $(0,1)$ and $(0,-1)$ are saddle points, the point $(2/3,1/3)$ is a local max point, and the point $(-2/3,-1/3)$ is a local min point.

From $f(x,0)=-x^3+x+1$ we see that f is unbounded both from below and above, so there is no global min or max.

Q-4) Maximize the function $f(x, y) = xy$ subject to the condition $\frac{x^2}{12} + \frac{y^2}{3} = 2$.

Solution:

Letting $g(x, y) = \frac{x^2}{12} + \frac{y^2}{3} - 2$, we solve simultaneously $\nabla f = \lambda \nabla g$ and $g = 0$, i.e. we solve

(1)..... $x = \frac{2\lambda y}{3}$.

(2)..... $y = \frac{2\lambda x}{12}$.

(3)..... $\frac{x^2}{12} + \frac{y^2}{3} = 2$.

First we see from both (1) and (2) that if $x = 0$, then $y = 0$, but this does not satisfy (3). So from (1) we solve for λ, substitute into (2) to obtain $\frac{x^2}{12} = \frac{y^2}{3}$. Putting this into (3) we get $x^2 = 12$, which gives $y^2 = 3$. Then at these critical points $xy = \pm 6$.

Since the function f clearly has a min and max (it is a continuous function on a closed and bounded set in $\mathbb{R}^2$), we conclude that min f is -6, and max of f is 6.
