

MATH 113 Solutions for Quiz 2

12 November 2003 Wednesday

Find the indicated derivatives. Do not simplify.

1) $f(x) = \frac{\sin x^2 - \cos^2 x}{\sqrt{1+x^2}}$. $f'(x) = ?$

$$f'(x) = \frac{(\cos x^2 \cdot 2x - 2 \cos x \cdot (-\sin x))(\sqrt{1+x^2}) - (\sin x^2 - \cos^2 x) \cdot \frac{x}{\sqrt{1+x^2}}}{1+x^2}.$$

2) $f(x) = \frac{x^2+1}{2x}$, $x(t) = t + \sin^2 t$. $\frac{df}{dt} = ?$

$$\frac{df}{dt} = \frac{df}{dx} \frac{dx}{dt} = \left(\frac{(2x)(2x) - (x^2+1)(2)}{4x^2} \right) (1 + 2 \sin t \cdot \cos t).$$

3) $f(x) = \sin x + \cos^3 x$, $x = (1+t+t^2)^{1/3}$, $t = \cos u + \sin u$. $\frac{df}{du} = ?$

$$\frac{df}{du} = \frac{df}{dx} \frac{dx}{dt} \frac{dt}{du} = (\cos x + 3 \cos^2 x \cdot (-\sin x)) \left(\frac{1}{3} (1+t+t^2)^{-2/3} \cdot (1+2t) \right) (-\sin u + \cos u).$$

3) $f(x) = \sin \left(\cos \left(\sin \left(\frac{x+1}{x-1} \right) \right) \right)$. $f'(x) = ?$

$$f'(x) = \cos \left(\cos \left(\sin \left(\frac{x+1}{x-1} \right) \right) \right) \cdot \left(-\sin \left(\sin \left(\frac{x+1}{x-1} \right) \right) \right) \cdot \left(\cos \frac{x+1}{x-1} \right) \cdot \left(\frac{1 \cdot (x-1) - (x+1) \cdot 1}{(x-1)^2} \right).$$