MATH 113 Solutions manual for Homework III

1. Find $f'(x)$.
a) Page 167 Exercise 6:

$$f(x) = \frac{1}{x+1} \Rightarrow f'(x) = -\frac{1}{(x+1)^2}.$$

b) Page 167 Exercise 9:

$$f(x) = \frac{1}{2+\cos x} \Rightarrow f'(x) = -\frac{1}{(2+\cos x)^2} \cdot (-\sin x) = \frac{\sin x}{(2+\cos x)^2}.$$

c) Page 167 Exercise 10:

$$\begin{aligned} f(x) = \frac{x^2+3x+2}{x^4+x^2+1} \Rightarrow f'(x) &= \frac{(2x+3)(x^4+x^2+1)-(x^2+3x+2)(4x^3+2x)}{x^4+x^2+1)^2} \\ &= \frac{-2x^5-9x^4-8x^3-3x^2-2x+3}{(x^4+x^2+1)^2}. \end{aligned}$$

d) Page 167 Exercise 11:

$$f(x) = \frac{2-\sin x}{2-\cos x} \Rightarrow f'(x) = \frac{(-\cos x)(2-\cos x)-(2-\sin x)\sin x}{(2-\cos x)^2} = \frac{-2\cos x - 2\sin x + 1}{(2-\cos x)^2}.$$

2. Page 173 Exercise 1. $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 1$. "Tangent line is horizontal" means $f'(x) = 0$. $f'(x) = x^2 - 4x + 3$ and $f'(x) = 0$ implies $x = 1$ and $x = 3$. So the points on the graph are $(1, -\frac{5}{3})$, $(3, 10)$.

3. Page 167 Exercise 5. $f(x) = x^2 + ax + b$, $g(x) = x^3 - c$. "Graphs of f and g intersect at $(1, 2)$" means $f(1) = g(1)$ and $g(1) = 2$. "Graphs of f and g have the same tangent line at $(1, 2)$" implies $f'(1) = g'(1)$. So we have three equations

$$\begin{aligned} f(1) = g(1) &\Rightarrow 1 + a + b = 1 - c \\ g(1) = 2 &\Rightarrow 1 - c = 2 \\ f'(1) = g'(1) &\Rightarrow 2 + a = 3 \end{aligned}$$

Solving these three equations we get: $a = 1,\ b = 0,\ c = -1$.