Math 113 Homework 4
Due: 19 November 2004 Friday class hour.
Instructor: Ali Sinan Sertöz

**Q-1) Exercise 12 on page 195.** Given a right circular cone with radius $R$ and altitude $H$. Find the radius and altitude of the right circular cylinder of largest lateral surface area that can be inscribed in the cone.

**Solution:** For the inscribed right cylinder let $r$ be the radius and $h$ the height. From similar triangles we obtain $\dfrac{R}{H} = \dfrac{R-r}{h}$, from where we find $h = H - \dfrac{H}{R}r$. The lateral area of the cylinder is $A = 2\pi rh$. Putting in the value of $h$ in terms of $r$ we get $A(r) = 2\pi r(H - \dfrac{H}{R}r)$, where the function makes sense for $0 \leq r \leq R$. We find that $f'(x) = 0$ for $r = R/2$. At the end points of the interval $f$ is zero, and in between $f$ is positive. Therefore $r = R/2$ gives the maximum value of $f$. The corresponding value for $h$ is then calculated to be $H/2$.

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**Q-2) Exercise 13 on page 195.** Find the dimensions of the right circular cylinder of maximum volume that can be inscribed in a right circular cone of radius $R$ and and altitude $H$.

**Solution:** Using the notation of the previous problem, the volume of the cylinder is $V = \pi r^2 h = \pi r^2 (H - \dfrac{H}{R}r)$, where $0 \leq r \leq R$. We find $V'(r) = 0$ for $r = 2R/3$ which, as before, gives the maximum value. The corresponding value of $h$ is $H/3$.

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**Q-3) Exercise 17 on page 195.** An open box is made from a rectangular piece of material by removing equal squares at each corner and turning up the sides. Find the dimensions of the box of largest volume that can be made in this manner if the material has sides (a) 10 and 10; (b) 12 and 18.

**Solution-a:** Let $x$ be the side of the square which is cut off. The function to maximize is $V(x) = x(10-2x)(10-2x)$, $0 \leq x \leq 5$. We find that $V'(x) = 12x^2 - 80x + 100 = 0$ when $x = 5$ or $x = 5/3$. At the end points $V$ is zero, so $V$ becomes maximum at $x = 5/3$. The other dimensions are $10 - 2x = 20/3$ each.

**Solution-b:** This time the function to maximize is $V(x) = x(12-2x)(18-2x)$, $0 \leq x \leq 6$. We find that $V'(x) = 12x^2 - 120x + 216 = 0$ when $x = 5 \pm \sqrt{7}$. But $5 + \sqrt{7}$ is outside the domain, so the only relevant root is $5 - \sqrt{7}$. Again at the end points the function is zero, so $x = 5 - \sqrt{7}$ gives the maximum value. The other dimensions are $12 - 2x = 2 + 2\sqrt{7}$ and $18 - 2x = 8 + 2\sqrt{7}$.

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**Q-4) Exercise 26 on page 196.** If $x > 0$, let $f(x) = 5x^2 + Ax^{-5}$, where $A$ is a positive constant. Find the smallest $A$ such that $f(x) \geq 24$ for all $x > 0$.

**Solution:** We want to find $A$ such that the minimum of $f$ is 24. We find that $f'(x) = (2x^7 - A)(5/x^6) = 0$ when $x = (A/2)^{1/7}$. Note that $f$ goes to infinity as $x$ goes to zero from the right and also as $x$ goes to infinity, i.e. $f$ goes to infinity as $x$ approaches to the end points of the domain. Therefore $f$ takes its minimum at the only zero of $f'$. We now solve $f((A/2)^{1/7}) = 24$ to find $A = 2(24/7)^{7/2}$. Since $f$ increases as $A$ increases, this must be the minimum value of $A$ making $f \geq 24$.