Math 113 Homework 5 • Due: 10 December 2004 Friday class hour. • Instructor: Sertöz

Q-1) Evaluate the following integral: $\int \cos x^{1/3}\, dx$.

Solution: Let $u = x^{1/3}$ or equivalently $u^3 = x$ and $dx = 3u^2\, du$. The integral then becomes $3\int u^2 \cos u\, du$, which can be evaluated by repeated use of by parts or by using tabular integration. Substituting back $u = x^{1/3}$ we find

$$\int \cos x^{1/3}\, dx = 3x^{2/3} \sin x^{1/3} + 6x^{1/3} \cos x^{1/3} - 6 \sin x^{1/3} + C$$

Q-2) Evaluate the following integral: $\int e^x \sin x\, dx$.

Solution: Integration by parts first gives $\int e^x \sin x\, dx = e^x \sin x - \int e^x \cos x\, dx$. Using integration by parts again with the second integral gives $\int e^x \sin x\, dx = e^x \sin x - e^x \cos x - \int e^x \sin x\, dx$. Solving for our integral from this we find

$$\int e^x \sin x\, dx = \frac{1}{2} e^x (\sin x - \cos x) + C$$

Q-3) Evaluate the following integral: $\int \sec^4 x\, dx$.

Solution: Start with integration by parts with $u = \sec^2 x$ to get $\int \sec^4 x\, dx = \sec^2 x \tan x - 2\int \sec^2 x \tan^2 x\, dx$. Now $\int \sec^2 x \tan^2 x\, dx = \int \sec^2 x (\sec^2 x - 1)\, dx = \int \sec^4 x\, dx - \int \sec^2 x\, dx = \int \sec^4 x\, dx - \tan x$. Putting this in and solving for the integral gives

$$\int \sec^4 x\, dx = \frac{1}{3} \sec^2 x \tan x + \frac{2}{3} \tan x + C.$$

Q-4) Differentiate the following function: $f(x) = \ln(x^2 + x + 1) + 3^{\cos x} + x^{x^x}$.

Solution: The key here is to write $3^{\cos x} = \exp(\cos x \ln 3)$, and $x^{x^x} = \exp(x^x \ln x) = \exp\left([\exp(x \ln x)] \ln x\right)$. The derivative then becomes

$$f'(x) = \frac{2x+1}{x^2+x+1} - 3^{\cos(x)} \sin(x) \ln(3) + x^{x^x} \left(x^x (\ln(x) + 1) \ln(x) + \frac{x^x}{x} \right)$$
