Calculus 113 Homework 6

Due date: 10 December 2007 Monday

Please take your homework solutions to room SA144, Ali Adalı's office until 17:00.

Q-1) Find a recursive formula for $I_{m,n} = \displaystyle\int x^m \ln^n x \ dx$, and use it to evaluate $\displaystyle\int_1^2 x^3 \ln^2 x \, dx$.

Q-2) Calculate $f'(1)$ where $f(x) = x^{x^x} + x^{\ln(1+x)}$.

Q-3) Show that for any real $x > 0$ and any integer $n > 0$, $\left(1 + \dfrac{x}{n}\right)^n < e^x$.

Q-4) Let $f(x)$ be the inverse hyperbolic sine function, *i.e.* $f(x) = \operatorname{arcsinh} x$. Find $f'(x)$ in terms of x in two different ways:

i) Start with "$y = \operatorname{arcsinh} x$ if and only if $x = \sinh y$", and then use chain rule.

ii) From the definition of $x = \sinh y$, solve explicitly for y in terms of x and then take derivatives. You will have to choose between two roots of a quadratic equation. Explain how you make your choice.

Q-5) Evaluate $\displaystyle\int \frac{31 - 14x}{3x^2 + 2x + 1} \, dx$.

Q-6) Let $f : (a, b) \longrightarrow \mathbb{R}$ be a uniformly continuous function. Show that there exists a continuous function $F : [a, b] \longrightarrow \mathbb{R}$ such that $F(x) = f(x)$ for all $x \in (a, b)$.

Please forward any comments or questions to `sertoz@bilkent.edu.tr`