Math 113 Calculus – Homework 1

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Please do not write anything inside the above boxes!

Check that there are 5 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-1) Let \( f : (a, b) \rightarrow \mathbb{R} \) be a differentiable function. Assume that for some \( x_0 \in (a, b) \), \( \lim_{x \to x_0} f'(x) \) exists and is \( L \). Show that \( f'(x_0) = L \).
Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a differentiable function. Assume that $f'$ is not continuous at some $x_0 \in \mathbb{R}$.

Prove or disprove each of the following statements:

(i) It is possible that $\lim_{x \to x_0^+} f'(x) = f'(x_0)$.

(ii) It is possible that $\lim_{x \to x_0^+} f'(x) = L \neq f'(x_0)$.

(iii) It is possible that $\lim_{x \to x_0^+} f'(x) = \infty$. 


Q-3) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function. Assume that $f'(x_0) > 0$ for some $x_0 \in \mathbb{R}$.

Prove or disprove the following statement:

There exists a $\delta > 0$ such that $f$ is increasing (strictly or not) on the interval $(x_0 - \delta, x_0 + \delta)$. 
Q-4) Find all the points, if any exist, on this ellipse
\[ \frac{(x-2)^2}{9} + \frac{(y-3)^2}{4} = 1 \]
satisfying the property that the line joining the point to the origin is tangent to the ellipse at that point.

(You may use a computer algebra program if need arises.)
Q-5) Find the equation of the tangent line to the curve \( x^2 y^3 - x^3 y^2 = 4 \) at the point \((1, 2)\). Show that there is no point \( p = (x_0, y_0) \) on the curve where the tangent line to the curve at \( p \) passes also from the origin.