Date: 20 January 2012, Friday
Time: 9:00-11:00
Ali Sinan Sertöz

NAME:..

STUDENT NO:..

Math 113 Calculus – Makeup Exam – Solutions

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

Check that there are 5 questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. **Every mathematical symbol and every equation you write must be part of a well constructed sentence. I will not read any hanging equations or symbols. I will not try to interpret your symbols. I will only grade what is written on your paper; I do not specialize in mind reading.**

NAME: STUDENT NO:

Q-1) Assuming that a continuous function on a closed and bounded interval is bounded, prove that it takes its minimum and maximum on that interval. Explain where you use the conditions *closed* and *bounded*.

Solution:

Let $f : [a, b] \to \mathbb{R}$ be a continuous function. Since we are assuming that f is bounded, its range is a bounded subset of $\mathbb{R}$ and has a supremum M, and an infimum m. Assume f never takes M on $[a, b]$. Then the function

$$g(x) = \frac{1}{M - f(x)}$$

is well defined and continuous on $[a, b]$, hence is bounded there, say

$$0 \leq g(x) \leq K,$$

for some $K > 0$. This however gives

$$f(x) < M - \frac{1}{K},$$

which contradicts the fact that M was supremum of the values of f. Hence, f must take M somewhere on $[a, b]$.

The minimum case is exactly the same.

The *closed* and *bounded* conditions are used in the proof of the theorem which says that a continuous function is bounded.

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Q-2) Write your answers to the space provided. No partial credits.

- $f(x) = (\cos x)^x + x^{\cos x}$, $f'(x) = (\cos x)^x[\ln\cos x + x\tan x] + x^{\cos x}[-\sin x \ln x + \cos x/x]$.
- $f(x) = x^3 - 7^x + x^x$, $f'(x) = 3x^2 - 7^x \ln 7 + x^x[\ln x + 1]$.
- $f(x) = \arctan(x + \ln(x))$, $f'(x) = \frac{1+1/x}{1+(x+\ln x)^2}$.
- $f(x) = \displaystyle\int_{x^3}^{x^5} \arcsin t^7 \, dt$, $f'(x) = 5x^4 \arcsin x^{35} - 3x^2 \arcsin x^{21}$.

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NAME: STUDENT NO:

Q-3) Write your answers to the space provided. No partial credits.

- $\displaystyle\int x\sin(5x^2)\,dx = -\frac{1}{10}\cos(5x^2) + C.$

- $\displaystyle\int x(\ln x)^2\,dx = \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{x^2}{4} + C.$

- $\displaystyle\int \frac{x}{(1+x)(1+x^2)}\,dx = -\frac{1}{2}\ln(1+x) + \frac{1}{4}\ln(1+x^2) + \frac{1}{2}\arctan x + C.$

- $\displaystyle\int x^2\sqrt{(e+5x^3)}\,dx = \frac{2}{45}(e+5x^3)^{3/2} + C.$

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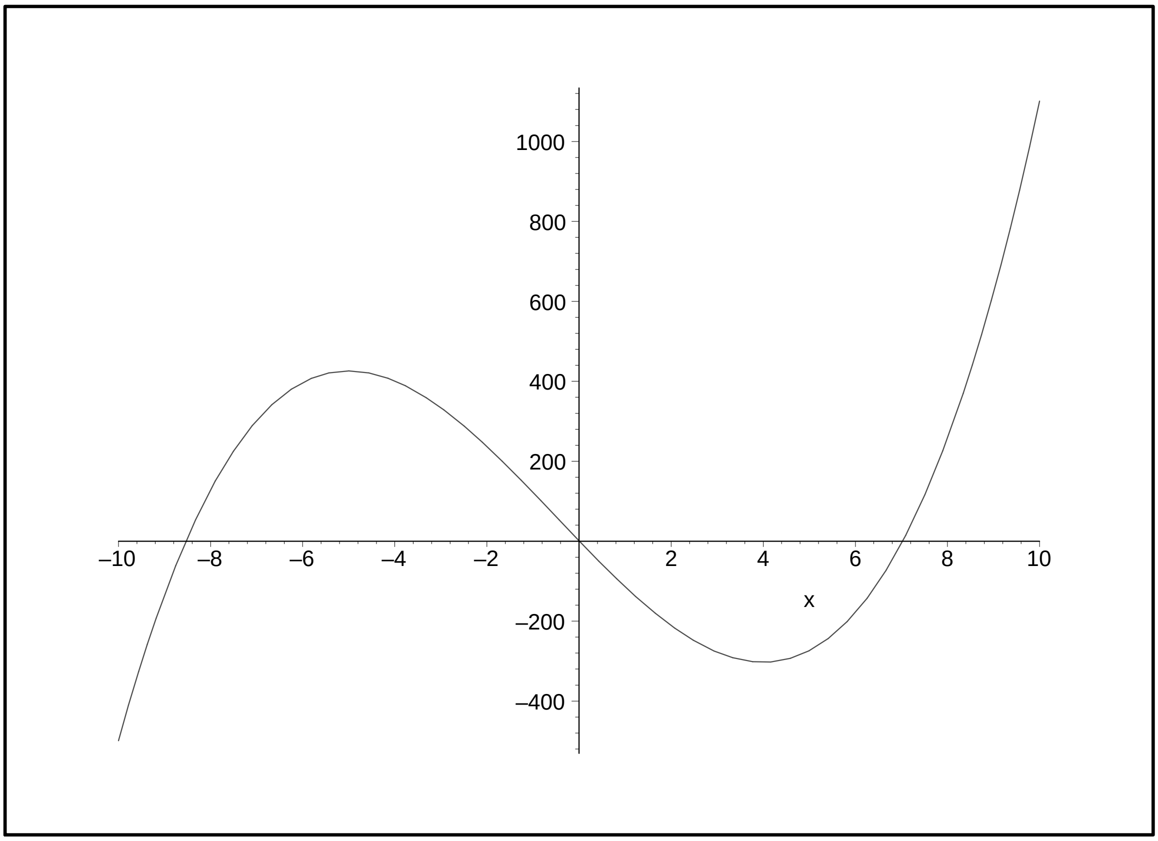
NAME: STUDENT NO:

Q-4) Plot the graph of $f(x) = 2x^3 + 3x^2 - 120x + 1$.

Solution:

$f'(x) = 6(x-4)(x+5)$, $f''(x) = 6(2x+1)$.



NAME: STUDENT NO:

Q-5) Let $D = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 + 2y = 1 \text{ and } x, y \geq 0\}$. Revolve the region D around y-axis and find the volume of the solid so obtained.

Solution:

Slice method: $\pi \displaystyle\int_0^{\sqrt{2}-1} (1 - 2y - y^2)\,dy = \pi \left(\frac{4\sqrt{2} - 5}{3}\right) \approx 0.68.$

Cylindrical shell method: $2\pi \displaystyle\int_0^1 (x\sqrt{2 - x} - x)\,dx = \pi \left(\frac{4\sqrt{2} - 5}{3}\right) \approx 0.68.$