DUE: 4 March 2005

MATH 114
HOMEWORK 2 – Solutions

For this homework set $\mathbf{p_1} = (1, 2, 3)$, $\mathbf{p_2} = (3, 2, 1)$, $\mathbf{p_3} = (4, 7, 5)$, $\mathbf{p_4} = (3, 1, -2)$.

In particular let $\mathbf{p} = p_2 - p_1 = (2, 0, -2)$, $\mathbf{q} = p_3 - p_1 = (3, 5, 2)$ and
$\mathbf{r} = p_4 - p_1 = (2, -1, -5)$.

Page numberings below refer to the Alternate Edition of Thomas' Calculus.

1.a Find an equation for the plane passing through the points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$.

$\mathbf{p} \times \mathbf{q} = (10, -10, 10) = 10(1, -1, 1)$, and $\mathbf{s} = (1, -1, 1)$ is a direction orthogonal to this plane. $\mathbf{s} \cdot \mathbf{p} = 2$, so $x - y + z = 2$ is an equation for this plane.

(see p824.)

1.b Find the area of the triangle formed by the points $\mathbf{p_1}$, $\mathbf{p_2}$, and $\mathbf{p_3}$.

This area is $\frac{1}{2}|\mathbf{p} \times \mathbf{q}| = \frac{1}{2}|(10, -10, 10)| = \sqrt{3}\, 5$.

(see p816.)

1.c Find the volume of the parallelepiped formed by the points $\mathbf{p_1}$, $\mathbf{p_2}$, $\mathbf{p_3}$ and $\mathbf{p_4}$. Is $\mathbf{p_1}$ in the plane formed by the points $\mathbf{p_2}$, $\mathbf{p_3}$ and $\mathbf{p_4}$?

This volume is $|\mathbf{r} \cdot (\mathbf{p} \times \mathbf{q})| = 20$. If the given points were in the same plane then this parallelepiped would have zero volume. So they are not in the same plane.

(see p819.)

2.a Find the distance from the point $\mathbf{p_3}$ to the line passing through the points $\mathbf{p_1}$ and $\mathbf{p_2}$.

This distance is $\dfrac{|\mathbf{p} \times \mathbf{q}|}{|\mathbf{p}|} = \dfrac{5\sqrt{3}}{\sqrt{2}}$.

(see p823.)

2.b Find the distance from the point $\mathbf{p_4}$ to the plane passing through the points $\mathbf{p_1}$, $\mathbf{p_2}$ and $\mathbf{p_3}$.

This distance is $\dfrac{|\mathbf{r} \cdot (\mathbf{p} \times \mathbf{q})|}{|\mathbf{p} \times \mathbf{q}|} = \dfrac{2}{\sqrt{3}}$.

(see p825.)

(see also the exercises on p852 for alternate formulas for distances.)