February 15, 2005

MATH 114 Homework 1

Turn in by February 22, 2005 until 10:30.

1. Evaluate the following improper integral:

$$\int_0^{+\infty} x^2 e^{-x}\, dx.$$

2. For a certain value of the constant C the following improper integral converges. Determine C and evaluate the integral.

$$\int_2^{+\infty} \left(\frac{Cx}{x^2+1} - \frac{1}{2x+1} \right) dx.$$

3. a) Find the function $f(x)$ such that

$$f(x) = \sum_{n=1}^{+\infty} n^2 x^n = x + 2^2x^2 + 3^2x^3 + 4^2x^4 + \cdots, -1 < x < 1$$

b) Find the following sum:

$$-\frac{1}{2} + \frac{2^2}{2^2} - \frac{3^2}{2^3} + \frac{4^2}{2^4} - \frac{5^2}{2^5} + \cdots$$

4. Use the identity $\cos^2 x = (1+\cos 2x)/2$ to obtain the Maclaurin series for $\cos^2 x$. Then differentiate this series to obtain the Maclaurin series for $-2\sin x \cos x$. Check that this is the series for $-\sin 2x$.