

**MATH 114 Homework 3**

Turn in by March 22, 2005 until 10:40.

1. Let

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

By using  $\varepsilon$ - $\delta$  definition of limit, show that  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$ .

2. Find  $\lim_{(x,y) \rightarrow (0,0)} \arctan\left(\frac{x-y}{x^2+y^2}\right)$  or show that the limit doesn't exist.

3. For the following functions verify that  $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$ .

a)  $f(x, y) = xy^2 + x^2y^3 + x^3y^4$

b)  $f(x, y) = x^y, x > 0$ .

4. Let  $f(u, v)$  be a differentiable function such that

$$\frac{\partial f}{\partial u}(2, 3) = -1, \quad \frac{\partial f}{\partial u}(12, 1) = -3, \quad \frac{\partial f}{\partial v}(2, 3) = 4, \quad \frac{\partial f}{\partial v}(12, 1) = 2.$$

Let  $z = f(x^2y, x^2 - y)$ . Find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  at the point  $(x, y) = (2, 3)$ .