Due on March 13, 2006, Monday, Class time. No late submissions!

MATH 114 Homework 4 – Solutions

1: Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^5}{x^2+y^4}$, if it exists.

Solution: $\displaystyle 0 \le \left|\frac{xy^5}{x^2+y^4}\right| = |xy|\,\frac{y^4}{x^2+y^4} \le |xy|\,\frac{x^2+y^4}{x^2+y^4} = |xy| \to 0$ as $(x,y)\to(0,0)$. Hence the required limit is zero by the sandwich theorem.

2: Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$, if it exists.

Solution: Let $y=\lambda x$. Then $\displaystyle\frac{xy}{x^2+y^2} = \frac{\lambda}{1+\lambda^2}$, so the limit does not exist.

3: Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+3y^4}$,if it exists.

Solution: Let $x=t^2$ and $y=\lambda t$. Then $\displaystyle\frac{xy^2}{x^2+3y^4} = \frac{\lambda^2t^4}{t^4+3\lambda^4t^4} = \frac{\lambda^2}{1+3\lambda^4}$ which depends on λ, i.e. the limit depends on the line of approach. Hence the limit does not exist.

4: Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^5-y^5}{(x^2+y^2)^2}$, if it exists.

Solution: Putting $x=r\cos\theta$ and $y=\sin\theta$ we find that the function becomes $r(\cos^5\theta-\sin^5\theta)$ and the limit as $r\to 0$ is zero:

5: Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^8}{x^4+y^6}$,if it exists.

Solution: $\displaystyle 0 \le \frac{x^2y^8}{x^4+y^6} < \frac{x^2y^6}{x^4+y^6} \le \frac{x^2(x^4+y^6)}{x^4+y^6} = x^2$ when $0<y<1$. Then by the sandwich theorem the limit is zero.
