

Due on March 20, 2006, Monday, Class time. No late submissions!

MATH 114 Homework 5

- 1: Let  $f(x, y) = \sin \ln(x^2 + y^2)$  where  $x = \cos \theta$  and  $y = 4 \sin \theta$ . Find  $\left. \frac{\partial f}{\partial \theta} \right|_{\theta=\pi/4}$ .
- 2: Let  $x^2 - xy + yz^3 + x^2z^2 - 2xy^3 = 0$  define  $z$  as a function of  $x$  and  $y$ . Find the equation of the tangent plane to this surface at the point  $(1, 1, 1)$ .
- 3: Consider the equations  $w = x^4 + 3x^2y + xy^2 + y^3$ ,  $x = s^2 + t^2$ ,  $y = \cos\left(\frac{5\pi}{t^2+1}\right)$ ,  $s = u + 2v$  and  $t = 3u + 4v$ . Find  $\left. \frac{\partial w}{\partial u} \right|_{(u,v)=(1,0)}$ .
- 4: Find the directional derivative of  $f(x, y, z) = 2x^2 + 3y^3 + 4z^4$  at the point  $(1, 2, 3)$  in the direction of  $(4, 5, 6)$ .
- 5: Assume that  $f(x, y) = 0$  defines a plane curve. Show that the gradient  $\nabla f$  is orthogonal to the tangent line of the curve at every point where the curve is smooth.