Due on April 3, 2006, Monday.

MATH 114 Homework 7 – Solutions

1: Sketch the region of integration, reverse the order of integration and then evaluate the integral

$$\int_0^1 \int_{1-y}^{\sqrt{1-y}} dxdy.$$

Solution:

$$\begin{aligned} \int_0^1 \int_{1-y}^{\sqrt{1-y}} dxdy &= \int_0^1 \int_{1-x}^{1-x^2} dydx \\ &= \int_0^1 (x - x^2)\, dx = \frac{1}{6} \cdot \end{aligned}$$

2: Sketch the region of integration, reverse the order of integration and then evaluate the integral

$$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} dxdy + \int_4^8 \int_{-\sqrt{8-y}}^{\sqrt{8-y}} dxdy.$$

Solution:

$$\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} dxdy + \int_4^8 \int_{-\sqrt{8-y}}^{\sqrt{8-y}} dxdy = \int_{-2}^2 \int_{x^2}^{8-x^2} dydx = \frac{64}{3} \cdot$$

3: Evaluate the integral

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx.$$

Solution:

$$\begin{aligned} \int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx &= \int_0^4 \int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y}\, dydx \\ &= \frac{1}{2} \int_0^4 e^{2y}\, dy \\ &= \frac{1}{4}\left(e^8 - 1\right) \cdot \end{aligned}$$

4: Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 25$ and the plane $x + z = 7$.

Solution:

$$\text{Volume } = \int_0^5 \int_0^{\sqrt{25-y^2}} (7-x)\, dxdy = \frac{175\pi}{4} - \frac{125}{3} \,.$$

5: Evaluate the integral

$$\int_0^1 \int_1^2 \frac{dxdy}{x+y} + \int_1^4 \int_{\sqrt{y}}^2 \frac{dxdy}{x+y}.$$

Solution:

$$\begin{aligned}
\int_0^1 \int_1^2 \frac{dxdy}{x+y} + \int_1^4 \int_{\sqrt{y}}^2 \frac{dxdy}{x+y} &= \int_1^2 \int_0^{x^2} \frac{dydx}{x+y} \\
&= \int_1^2 \left(\ln(x+y)\Big|_{y=0}^{y=x^2} \right) dx \\
&= \int_1^2 \ln(1+x)\, dx \\
&= \left((1+x)\ln(1+x) - (1+x)\Big|_1^2 \right) = \ln\frac{27}{4} - 1.
\end{aligned}$$

Send comments and corrections to `sertoz@bilkent.edu.tr` please.