Due on May 9, 2006, Tuesday, Class time. No late submissions!

MATH 114 Homework 9 – Last one :-)

1: Intersect the sphere $x^2 + y^2 + z^2 = 196$ with the cylindrical surface $x^2 + y^2 = 14y$, $z \geq 0$, and calculate **(i)** the area of the spherical cap so formed and **(ii)** the volume under this cap and over the xy-plane.

2: Find the area of that portion of the sphere $x^2 + y^2 + z^2 = 4$ lying between the planes $z = \sqrt{3}$ and $z = -1$.

3: Find an equation for the plane through the origin such that the circulation of the flow $\mathbf{F} = z\ \mathbf{i} + x\ \mathbf{j} + y\ \mathbf{k}$ around the circle C of intersection of the plane with the sphere $x^2 + y^2 + z^2 = 4$ is a maximum. Recall that the circulation of the flow $\mathbf{F}$ around the circle C is given by $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{r}$ is a smooth parametrization of C. What happens if we replace the radius of the sphere by some other value?

4: Calculate the area of the region on the Earth bounded by the meridians 120° and 150° west longitude and the circles 30° and 45° north latitude, assuming that the Earth is spherical with radius R km.

5: Find the outward flux of the vector field $\mathbf{F} = x^2\ \mathbf{i} + y^2\ \mathbf{j} + z^2\ \mathbf{k}$ across the boundary ∂D of the cube D cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$. The outward flux of $\mathbf{F}$ on ∂D is given by the integral $\int\int_{\partial D} \mathbf{F} \cdot \mathbf{n}\ d\sigma$, where $\mathbf{n}$ is the unit outward normal on the faces of ∂D.