Date: 30 May 2011, Monday
Time: 10:00-12:00
Ali Sinan Sertöz

NAME:..

STUDENT NO:..

Math 114 Calculus II – Final Exam Make-Up – Solutions

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

Check that there are 5 questions on your exam booklet. **Write your name on top of every page.** Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. **Also note that** if you write down something which you don't believe yourself, the chances are that I will not believe it either.

NAME: STUDENT NO:

Q-1) Check if the following series converge or diverge:

$$\textbf{a)} \sum_{n=1}^{\infty} \sin \frac{1}{n!} \quad \text{and} \quad \textbf{b)} \sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}.$$

Solution:

We know that $\sum 1/n!$ converges. Use limit comparison test.

$$\lim_{n\to\infty} \frac{\sin(1/n!)}{1/n!} = 1,$$

hence $\sum \sin(1/n!)$ converges.

For the second series use integral test.

$$\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \left(-\frac{1}{\ln x}\Big|_2^{\infty} \right) < \infty$$

so the series converges.

NAME: STUDENT NO:

Q-2) Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \left(\frac{x-y}{x^2+y^2}, \frac{x+y}{x^2+y^2}\right)$ and C is the square in the plane with vertices at the points $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$, traversed counterclockwise.

Solution:

Let $F = (M, N)$. Observe that $M_y = N_x$. So if C' is a circle of radius R centered at the origin with $0 < R < 1/\sqrt{2}$, then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_{C'} \mathbf{F} \cdot d\mathbf{r},$$

which follows from Green's theorem.

Now parametrizing C' as $\mathbf{r}(t) = (R\cos t, R\sin t)$ and substituting in, we find that $\mathbf{F} \cdot d\mathbf{r} = dt$. Hence the integral becomes 2π.

NAME: STUDENT NO:

Q-3) Find a normal vector and equations of the tangent plane and normal line to the graph $z = \sin(xy)$ at the point where $x = \pi/3$ and $y = -1$.

Solution:

The point on the graph is $(\pi/3, -1, -\sqrt{3}/2)$.

Tangent plane: $3x - \pi y + 6z = 2\pi - 3\sqrt{3}$.

The normal line: $\dfrac{6x - 2\pi}{-3} = \dfrac{6y + 6}{\pi} = \dfrac{6z + 3\sqrt{3}}{-6}$.

NAME: STUDENT NO:

Q-4) Find the volume of the region R lying below the plane $z = 3 - 2y$ and above the paraboloid $z = x^2 + y^2$.

Solution:

$$Volume = \int_{-3}^{1} \int_{-\sqrt{3-2y-y^2}}^{\sqrt{3-2y-y^2}} \int_{x^2+y^2}^{3-2y} dzdxdy = 8\pi.$$

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Q-5) Find the volume of the region lying inside all three of the circular cylinders

$$x^2 + y^2 = a^2, \quad x^2 + z^2 = a^2 \text{ and } y^2 + z^2 = a^2$$

where $a > 0$.

Solution:

After a careful sketching of the solid under question, we find that the volume is

$$\begin{aligned} V &= 16\left(\int_0^{a/\sqrt{2}} \int_0^x \sqrt{a^2 - x^2}\, dydx + \int_{a/\sqrt{2}}^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2}\, dydx\right) \\ &= 16\left(1 - \frac{1}{\sqrt{2}}\right) a^3. \end{aligned}$$