Due Date: March 2, 2011 Wednesday

Ali Sinan Sertöz

NAME:...

STUDENT NO:...

Math 114 Calculus – Homework 1

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are 4 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

Q-1) Assume that each $a_n > 0$ and $\displaystyle\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \rho$. Show that $\displaystyle\lim_{n \to \infty} \sqrt[n]{a_n} = \rho$.

Q-2) Define a function $f : \mathbb{R} \to \mathbb{R}$ as follows.

$$f(x) = \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0. \end{cases}$$

(i) Sketch the graph of $y = f(x)$.

(ii) Show that $f^{(n)}(0) = 0$ for all $n = 0, 1, 2, \ldots$.

(iii) Show that f is C^∞ but is not analytic at the origin.

Q-3) Let $f : [1, \infty) \to \mathbb{R}$ be an increasing function.

(i) Show that

$$f(1) + \cdots + f(n-1) < \int_1^n f(x)\, dx < f(2) + \cdots + f(n).$$

(ii) Choosing a suitable f, show that $\displaystyle\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{n} = \frac{1}{e}$.

(iii) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{e^n n!}{n^n}$ converge?

(iv) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{e^n n!}$ converge?

Q-4) Find the sum $1 - \dfrac{1}{4} + \dfrac{1}{7} - \dfrac{1}{10} + \cdots + \dfrac{(-1)^n}{1 + 3n} + \cdots$.