Due Date: May 2, 2011 Monday NAME:..

Ali Sinan Sertöz STUDENT NO:..

Math 114 Calculus – Homework 3

1	2	TOTAL
50	50	100

Please do not write anything inside the above boxes!

Check that there are 2 questions on your booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit.

NAME: STUDENT NO:

Q-1) Let C be a piecewise smooth curve in the xy-plane that does not pass through the origin. Let $\theta = \theta(x, y)$ be the polar angle coordinate of the point $P = (x, y)$ on C, not restricted to an interval of length 2π, but varying continuously as P moves from one end of C to the other end.

(a) Show that $\nabla\theta = -\dfrac{y}{x^2+y^2}\mathbf{i} + \dfrac{x}{x^2+y^2}\mathbf{j}$.

(b) Show that $\dfrac{1}{2\pi}\displaystyle\oint_C \frac{x\,dy - y\,dx}{x^2+y^2}$ is always an integer when C is a closed curve.

Solution:

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Q-2) A smooth surface S is given parametrically by

$$\mathbf{r} = (\cos 2u)(2 + v\cos u)\mathbf{i} + (\sin 2u)(2 + v\cos u)\mathbf{j} + v\sin u\mathbf{k}$$

where $0 \leq u \leq 2\pi$ and $-1 \leq v \leq 1$.

Show that for *every* smooth vector field $\mathbf{F}$ on S,

$$\iint_S \mathbf{F} \cdot \mathbf{N}\, dS = 0,$$

where $\mathbf{N} = \mathbf{N}(u, v)$ is a unit normal vector field on S that depends continuously on (u, v).

Solution: