Date: 25 May 2012, Friday
Time: 12:30-14:30
Ali Sinan Sertöz

NAME:..

STUDENT NO:..

Math 114 Calculus II – Final Exam – Solutions

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

PLEASE READ:
Check that there are 5 questions on your exam booklet. Write your name on the top of every page. Show your work in reasonable detail, if that is necessary. A correct answer without proper reasoning may not get any credit, except in the fill-in-the-blanks type problems.

NAME: STUDENT NO:

Q-1) Write your answers on the spaces provided. No partial credits.

$f(x) = \cos(\ln(e^x + 1))$	$f'(x) = -\sin(\ln(e^x+1))\dfrac{e^x}{(e^x+1)}$
$f(x) = (\sec x)^{\tan x}$	$f'(x) = (\sec x)^{\tan x}[\sec^2 x \ln \sec x + \tan^2 x]$
$f(x) = 2^x + 2^2 + x^2$	$f'(x) = 2^x \ln 2 + 2x$
$f(x) = \arctan(\cosh\ x)$	$f'(x) = \dfrac{\sinh x}{1+\cosh^2 x}$
$f(x,y) = x^7 + x^6y^5 + y + y^3$, $x(t) = 2\cos t + 3\sin t - t - 1$, $y(t) = 2e^t + 3\ln(1+t^2) + t^3 + t$, $h(t) = f(x(t), y(t))$.	$h'(0) = 677$

Nothing below this line will be read on this page!

NAME: STUDENT NO:

Q-2 Find the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}\, x^n.$$

Find also the convergence behavior of the series at the end points of its interval of convergence.
Hint: $n^{1/3} < \frac{e^n n!}{n^n} < 3n,$ *for all* $n = 1, 2, 3, \ldots.$

Solution:

Let $a_n(x) = \dfrac{n!}{n^n}\, x^n$. Apply ratio test to find

$$\left|\frac{a_{n+1}(x)}{a_n(x)}\right| = \frac{|x|}{(1+1/n)^n} \to \frac{|x|}{e} \text{ as } n \to \infty.$$

The series absolutely converges for $|x| < e$, hence the radius of convergence is e.

From the hint we know that $a_n(e) > n^{1/3}$, hence $\lim_{n\to\infty} a_n(e) = \lim_{n\to\infty} |a_n(-e)| = \infty$. Thus the series diverges at both end points, by divergence test.

Therefore the interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}\, x^n$ is $(-e,\, e)$.

NAME: STUDENT NO:

Q-3) Find all local/global minimum and maximum points of $f(x, y) = x^4 + 24y^2 - 4xy^3$, if they exist. Also find any saddle points if they exist.

Solution:

First find the critical points. $f_x = 0$ and $f_y = 0$ give $(0, 0)$, $(2, 2)$ and $(-2, -2)$ as critical points. The second derivative test immediately gives the points $(2, 2)$ and $(-2, -2)$ as saddle points. The discriminant vanishes at $(0, 0)$. Here we observe that

$$f(x, y) - f(0, 0) = x^4 + 4y^2(6 - xy) > 0$$

for $|x|, |y| < 1$. So the origin is a local minimum point.

The function however is not bounded since $\lim_{x\to\infty} f(x, 0) = \infty$ and $\lim_{y\to\infty} f(1, y) = -\infty$. So there is no global minimum or maximum.

NAME: STUDENT NO:

Q-4) Evaluate the following integrals.

$$(\mathbf{i}) \int_0^{\pi}\int_y^{\pi} \frac{\sin x}{x}\, dxdy, \text{ and } (\mathbf{ii}) \int_0^2\int_y^2 x^2\sin(xy)\, dxdy.$$

Solution:

(i)

$$\int_0^{\pi}\int_y^{\pi} \frac{\sin x}{x}\, dxdy = \int_0^{\pi}\int_0^{x} \frac{\sin x}{x}\, dydx = \int_0^{\pi} \sin x\, dx = 2.$$

(ii)

$$\begin{aligned}
\int_0^2\int_y^2 x^2\sin(xy)\, dxdy &= \int_0^2\int_0^x x^2\sin(xy)\, dydx \\
&= \int_0^2 \left(-x\cos(xy)\Big|_{y=0}^{y=x}\right)\, dx \\
&= \int_0^2 (-x\cos x^2 + x)\, dx \\
&= \frac{1}{2}\left(-\sin x^2 + x^2\Big|_0^2\right) \\
&= 2 - \frac{1}{2}\sin 4.
\end{aligned}$$

NAME: STUDENT NO:

Q-5) Let D be the region in the first octant lying under the cone $z = \sqrt{x^2+y^2}$, inside the cylinder $x^2+y^2=1$, and bounded by the coordinate planes. Write the limits of integration for the following iterated integrals for Cartesian, spherical and cylindrical coordinates respectively. Evaluate one of these integrals and write the answer in the box provided. No partial credits.
(Grading: each box=1 point, except the answer box which is 2 credits.)

$$\iiint_D (6+4y)\, dV = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} (6+4y)\, dz\, dy\, dx$$

$$= \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} (6+4\rho\sin\phi\sin\theta)(\rho^2\sin\phi)\, d\rho\, d\phi\, d\theta$$

$$= \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} (6r+4r^2\sin\theta)\, dz\, dr\, d\theta$$

$$= \boxed{}\,.$$

Solution:

$$\iiint_D (6+4y)\, dV = \int_{\boxed{0}}^{\boxed{1}} \int_{\boxed{0}}^{\boxed{\sqrt{1-x^2}}} \int_{\boxed{0}}^{\boxed{\sqrt{x^2+y^2}}} (6+4y)\, dz\, dy\, dx$$

$$= \int_{\boxed{0}}^{\boxed{\pi/2}} \int_{\boxed{\pi/4}}^{\boxed{\pi/2}} \int_{\boxed{0}}^{\boxed{1/\sin\phi}} (6+4\rho\sin\phi\sin\theta)(\rho^2\sin\phi)\, d\rho\, d\phi\, d\theta$$

$$= \int_{\boxed{0}}^{\boxed{\pi/2}} \int_{\boxed{0}}^{\boxed{1}} \int_{\boxed{0}}^{\boxed{r}} (6r+4r^2\sin\theta)\, dz\, dr\, d\theta$$

$$= \boxed{\pi+1}\,.$$