Due Date: March 2, 2012 Friday class time NAME:...

Ali Sinan Sertöz STUDENT NO:...

Math 114 Calculus – Homework 2

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are 4 questions on your booklet. Write your name on top of every page.
Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.
Everything you write on your paper should be part of a well constructed sentence. No hanging equations will be read. No sequence of equations will be read unless they are part of a well constructed, meaningful sentence.

Q-1) Fix a real number $a \geq 0$. For each such number define a function

$$f_a(x) = \left(1 + \frac{1}{x}\right)^{x+a}, \text{ for } x \geq 1.$$

(i) Show that $\lim_{x \to \infty} f_a(x) = e$ for all $a \geq 0$.

(ii) Show that, when $0 \leq a < 1/2$, $f_a(x)$ is strictly increasing on the interval $(a/(1-2a), \infty)$, and in particular $f_a(x) < e$ on this interval.

(iii) Show that, when $a > 1/2$, $f_a(x)$ is strictly decreasing, and in particular $f_a(x) > e$ for all $x \geq 1$.

Solution:

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Q-2) Using the results of Question-1, show that

$$n^{1/3} < \frac{e^n n!}{n^n} < 3n, \text{ for all } n = 1, 2, 3, \ldots.$$

Solution:

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Q-3) Using Stirling's formula, show that there exist real numbers $0 < \alpha < \beta$ such that

$$\alpha\, n^{1/2} < \frac{e^n n!}{n^n} < \beta\, n^{1/2},$$

for all sufficiently large n.

Solution:

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Q-4) Find the radius of convergence for each of the series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}\, x^n, \text{ and } \sum_{n=1}^{\infty} \frac{n^n}{n!}\, x^n.$$

Find also the convergence behaviors of each series at the end points of its interval of convergence. You will notice that the purpose of all the previous work in this homework was to be able to examine these end points.

(Grading for this problem: 1 point for each radius of convergence, 8 points for correctly examining the end points for the first series, and 15 points for the end points of the second series.)

Solution: