Due Date: March 30, 2012 Friday class time   NAME:..

Ali Sinan Sertöz   STUDENT NO:..

Math 114 Calculus – Homework 3

1	2	3	4	TOTAL
25	25	25	25	100

Please do not write anything inside the above boxes!

Check that there are 4 questions on your booklet. Write your name on top of every page.

Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.

Everything you write on your paper should be part of a well constructed sentence. No hanging equations will be read. No sequence of equations will be read unless they are part of a well constructed, meaningful sentence.

Q-1) Let $f(x, y)$ be a function defined in some open neighborhood of (x_0, y_0). Assume that there exist constants A and B such that f satisfies one of the following conditions DIFF1 or DIFF2.

$$\lim_{(h,k)\to(0,0)} \frac{f(x_0 + h, y_0 + k) - f(x_0, y_0) - Ah - Bk}{\sqrt{h^2 + k^2}} = 0. \qquad \text{(DIFF1)}$$

$$f(x_0 + h, y_0 + k) = f(x_0, y_0) + Ah + Bk + \epsilon_1 h + \epsilon_2 k, \qquad \text{(DIFF2)}$$

where ϵ_i is a function of h and k such that $\lim_{(h,k)\to(0,0)} \epsilon_i = 0$, $i = 1, 2$.

(i) Show that if f satisfies the condition DIFF1, then $f_x(x_0, y_0)$, $f_y(x_0, y_0)$ exist and $A = f_x(x_0, y_0)$, $B = f_y(x_0, y_0)$.

(ii) Show that if f satisfies the condition DIFF2, then $f_x(x_0, y_0)$, $f_y(x_0, y_0)$ exist and $A = f_x(x_0, y_0)$, $B = f_y(x_0, y_0)$.

(iii) Show that the conditions DIFF1 and DIFF2 are equivalent.

Remark: A function satisfying any of the equivalent conditions DIFF1 or DIFF2 is called differentiable at (x_0, y_0).

Solution:

Q-2) Define

$$f(x, y) = \begin{cases} \dfrac{y^5 - x^2 y}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

(i) Show that f is continuous at $(0, 0)$.

(ii) Show that $f_x(0, 0)$ and $f_y(0, 0)$ exist.

(iii) Show that f is not differentiable at $(0, 0)$.

Solution:

Q-3) Define

$$g(x, y) = \begin{cases} \dfrac{y^5 + x^2 y}{x^2 + y^4} & \text{if } (x, y) \neq 0, \\ 0 & \text{if } (x, y) = 0. \end{cases}$$

(i) Show that g is continuous at $(0, 0)$.

(ii) Show that $g_x(0, 0)$ and $g_y(0, 0)$ exist.

(iii) Show that g is differentiable at $(0, 0)$.

Solution:

Q-4) Define

$$h(x, y) = \begin{cases} \dfrac{x^3 y - x y^3}{x^2 + y^2} & \text{if } (x, y) \neq 0, \\ 0 & \text{if } (x, y) = 0. \end{cases}$$

(i) Find h_x, h_y, h_{xy}, h_{yx} at points $(x, y) \neq (0, 0)$.

(ii) Find h_x, h_y, h_{xy}, h_{yx} at points $(x, y) = (0, 0)$.

(iii) Did you get $h_{xy}(0, 0) = h_{yx}(0, 0)$? Explain why?.

Solution: