

Due Date: April 20, 2012 Friday class time

NAME:.....

Ali Sinan Sertöz

STUDENT NO:.....

**Math 114 Calculus – Homework 4**

1	2	3	4	TOTAL
25	25	25	25	100

*Please do not write anything inside the above boxes!*

Check that there are 4 questions on your booklet. Write your name on top of every page.

Show your work in reasonable detail. A correct answer without proper reasoning may not get any credit.

Everything you write on your paper should be part of a well constructed sentence. No hanging equations will be read. No sequence of equations will be read unless they are part of a well constructed, meaningful sentence.

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**Q-1)** Read Theorem 8 on page 731. Then examine Example 3 on page 738. Now show that the equation  $\frac{1 + x + y^3}{1 + x^3 + y^4} = 1$  has a solution of the form  $y = f(x)$  near  $x = 0$  satisfying  $f(0) = 1$ , and find the terms up to fifth degree for the Taylor series for  $f(x)$  in powers of  $x$

**Solution:**

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**Q-2)** Let  $f(x, y, z) = (x^2 + y^2) \ln(1 + y^2) + yz + xz^3$ . Let  $P_0$  be the point  $(1, 0, 2)$ .

- (i) Find the gradient of  $f$  at  $P_0$ .
- (ii) Find the linearization of  $f$  at  $P_0$ .
- (iii) Find the equation for the tangent plane at  $P_0$  to the level surface of  $f$  through  $P_0$ .
- (iv) If a bird flies through  $P_0$  with speed 5, heading directly toward the point  $(2, -1, 1)$ , what is the rate of change of  $f$  as seen by the bird as it passes through  $P_0$ ?
- (v) In what direction from  $P_0$  should the bird fly at speed 5 to experience the greatest rate of increase of  $f$ ?

**Solution:**

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**Q-3)** Find all local/global minimum and maximum points of  $f(x, y) = x^4 + 24y^2 - 4xy^3$ , if they exist.  
Also find any saddle points if they exist.

**Solution:**

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**Q-4)** Among all the ellipsoids of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

which pass through the point  $(2, 1, 3)$ , find the ones with the minimum and the maximum volumes, if they exist.

**Solution:**