

Summer 2007-08 MATH 116 Homework 3

Due on July 18.

No late homework will be accepted.

1. Find the outward flux of the vector field

$$\mathbf{F}(x, y) = (x^2 + y^2) \sin(x) \mathbf{i} + e^{xy^2} \ln\left(\frac{y}{e}\right) \mathbf{j}$$

across the rectangle with vertices $(0, 1)$, $(\pi, 1)$, $(0, e)$, and (π, e) .

2. Find the work done by the force

$$\mathbf{F}(x, y, z) = (2xz^3 + e^y) \mathbf{i} + (xe^y + 4y^3 \cos(z)) \mathbf{j} + (3x^2z^2 - y^4 \sin(z)) \mathbf{k}$$

over the curve parameterized by

$$\mathbf{r}(t) = \sin(\pi t) \mathbf{i} + t^3 \mathbf{j} + (2t - 1) \mathbf{k}, \quad \text{for } 0 \leq t \leq \frac{1}{2}$$

in the direction of increasing t .

3. Among all smooth simple closed curves in the plane oriented counterclockwise, find the one along which the work done by

$$\mathbf{F}(x, y) = \left(\frac{4}{3}y^3 - 20y + 5\right) \mathbf{i} + (1 + 5x - 3x^3) \mathbf{j}$$

is greatest and calculate the area of the region enclosed by this smooth simple closed curve.

4. Let C be a smooth curve that encloses a region R such that the area of the region R is 7π and the interior of the region R contains the rectangle

$$D = \{(x, y) \mid -1 \leq x \leq 1 \text{ and } -1 \leq y \leq 1\}.$$

Compute the outward flux of the vector field

$$\mathbf{F}(x, y) = \left(\frac{2x + y}{x^2 + y^2} + 3x + 6y\right) \mathbf{i} + \left(\frac{2y - x}{x^2 + y^2} + 5x + 7y\right) \mathbf{j}$$

across the curve C .

5. Find the area of the surface $z = 2xy$ inside the cylinder $x^2 + y^2 = 9$.