Due date: July 3, 2009, Friday

**Math 116 Calculus – Homework # 2 – Solutions**

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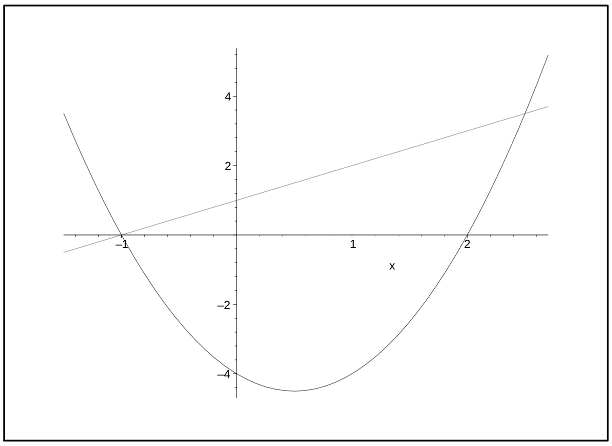
**Q-1)** Evaluate the double integral of the function $f(x, y) = xy$ over the region bounded by the curves $y = 2x^2 - 2x - 4$ and $y = x + 1$.

**Solution:**

$$\begin{aligned}\int\int_D xy\,dy\,dx &= \int_{-1}^{5/2}\int_{2x^2-2x-4}^{x+1} xy\,dy\,dx \\ &= \int_{-1}^{5/2}\left(-2\,x^5 + 4\,x^4 + (13/2)\,x^3 - 7\,x^2 - (15/2)\,x\right)\,dx \\ &= \frac{2401}{1920},\end{aligned}$$

where $D$ is the following region.

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**Q-2)** Let $R$ be the region in the first quadrant of the $xy$-plane bounded by the parabolas $y = x^2$, $y = x^2 + 1$, $y + x^2 = 4$ and $y + x^2 = 2$. Evaluate the double integral of $f(x, y) = e^{2y-x^2}\,x$ over $R$.

**Solution:** Let $T$ be the transformation from $xy$-plane to $uv$-plane given by $u = y - x^2$, $v = y + x^2$. Then $J(T) = -4x$, $2y - x^2 = (3/2)u + (1/2)v$, and $T(R)$ is the region in $uv$-plane bounded by the lines $u = 1$, $u = 2$, $v = 2$ and $v = 4$. Finally we have

$$\begin{aligned}\int\int_R e^{2y-x^2}\,x &= \int\int_{T(R)} e^{(3/2)u+(1/2)v}\,x\left|\frac{1}{J(T)}\right|\,du\,dv \\ &= \frac{1}{4}\left(\int_1^2 e^{(3/2)u}\,du\right)\left(\int_2^4 e^{v/2}\,dv\right) \\ &= \frac{1}{3}(e^3 - e^{3/2})(e^2 - e).\end{aligned}$$

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**Q-3)** Evaluate $\int_0^8 \int_{y^{1/3}}^2 \cos(x^2)\,dx\,dy$.

**Solution:**

$$\begin{aligned}
\int_0^8 \int_{y^{1/3}}^2 \cos(x^2)\,dx\,dy &= \int_0^2 \int_0^{x^3} \cos(x^2)\,dy\,dx \\
&= \int_0^2 x^3 \cos(x^2)\,dx \\
&\quad \text{use by parts with } u = x^2,\ dv = x \cos x^2 \text{ to get} \\
&= \left( \frac{1}{2} x^2 \sin x^2 \Big|_0^2 \right) - \int_0^2 x \sin x^2\,dx \\
&= 2 \sin 4 + \left( \frac{1}{2} \cos x^2 \Big|_0^2 \right) \\
&= 2 \sin 4 + \frac{1}{2} \cos 4 - \frac{1}{2}.
\end{aligned}$$

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**Q-4)** Find the volume bounded by the plane $8y + z = 12$ and the paraboloid $z = 4x^2 + 4y^2$.

**Solution:**

$$\begin{aligned}
\text{Volume} &= \int_{-2}^2 \int_{-1-\sqrt{4-x^2}}^{-1+\sqrt{4-x^2}} \int_{4x^2+4y^2}^{12-8y} dz\,dy\,dx \\
&= \frac{16}{3} \int_{-2}^2 (4 - x^2)^{3/2}\,dx \\
&\quad \text{Here put } x = 2\sin\theta \text{ to obtain} \\
&= \frac{256}{3} \int_{-\pi/2}^{\pi/2} \cos^4\theta\,d\theta \\
&= 32\,\pi.
\end{aligned}$$

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**Q-5)** Find the volume that lies inside the sphere $x^2 + y^2 + z^2 = 4$, above the $xy$-plane and under the cone $z^2 = x^2 + y^2$.

**Solution:**

$$\begin{aligned}
\text{Volume} &= \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^2 \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta \\
&= \left( \int_0^{2\pi} d\theta \right) \left( \int_{\pi/4}^{\pi/2} \sin\phi\,d\phi \right) \left( \int_0^2 \rho^2\,d\rho \right) \\
&= \frac{8\sqrt{2}\pi}{3}.
\end{aligned}$$

send questions and comments to sertoz@bilkent.edu.tr