Date: June 12, 2009, Friday   NAME:...

STUDENT NO:..

SECTION NUMBER: ....

Math 116 Calculus – QUIZ # 1

Q-1) Find the following limits, if they exist, and prove your results:

$$\text{(i)} \quad \lim_{(x,y)\to(0,0)} \frac{x^2 y^2}{x^6 + y^2}, \quad \text{(ii)} \quad \lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^6 + y^2}.$$

Solutions: For the first one observe that

$$0 \leq \frac{x^2 y^2}{x^6 + y^2} = x^2 \frac{y^2}{x^6 + y^2} \leq x^2,$$

and by the sandwich theorem, the limit is zero.

For the second one, try the path $y = \lambda x^2$:

$$\lim_{\substack{(x,y)\to(0,0) \\ y=\lambda x^2}} \frac{x^2 y}{x^6 + y^2} = \lim_{x\to 0} \frac{\lambda x^4}{x^6 + \lambda^2 x^4} = \lim_{x\to 0} \frac{\lambda}{x^2 + \lambda^2} = \frac{1}{\lambda}.$$

The limit on different paths do not agree so the limit does not exist.