Date: July 3, 2009, Friday

Math 116 Calculus – QUIZ # 6 – Solutions

Question: Evaluate the integral $\displaystyle\int\int_D \frac{\cos(x-y)}{x^2+2xy+y^2}\,dx\,dy$ where D is the region in the xy-plane bounded by the lines $x+y=1$, $x+y=2$, $x-y=3$ and $x-y=5$.

Solution: Use the substitution $u=x+y$, $v=x-y$.

Then $\displaystyle\frac{\partial(u,v)}{\partial(x,y)}=-2$, so $\displaystyle\frac{\partial(x,y)}{\partial(u,v)}=-\frac{1}{2}$.

In the uv-plane the region is bounded by the lines $u=1$, $u=2$, $v=3$ and $v=5$. Finally, the integral becomes:

$$\begin{aligned}\int\int_D \frac{\cos(x-y)}{x^2+2xy+y^2}\,dx\,dy &= \int_3^5\int_1^2 \frac{\cos v}{u^2}\left|-\frac{1}{2}\right|\,du\,dv \\ &= \frac{1}{2}\left(\sin v\Big|_3^5\right)\left(-\frac{1}{u}\Big|_1^2\right)=\frac{1}{4}(\sin 5-\sin 3) \\ &\approx -0.275.\end{aligned}$$

Question: Evaluate the integral $\displaystyle\int\int_D \frac{\sin(x-2y)}{4x^2+4xy+y^2}\,dx\,dy$ where D is the region in the xy-plane bounded by the lines $x-2y=1$, $x-2y=3$, $2x+y=4$ and $2x+y=10$.

Solution: Use the substitution $u=x-2y$, $v=2x+y$. Then $\displaystyle\frac{\partial(u,v)}{\partial(x,y)}=5$, so $\displaystyle\frac{\partial(x,y)}{\partial(u,v)}=\frac{1}{5}$.

In the uv-plane the region is bounded by the lines $u=1$, $u=3$, $v=4$ and $v=10$. Finally, the integral becomes:

$$\begin{aligned}\int\int_D \frac{\sin(x-2y)}{4x^2+4xy+y^2}\,dx\,dy &= \int_4^{10}\int_1^3 \frac{\sin u}{v^2}\left(\frac{1}{5}\right)\,du\,dv \\ &= \frac{1}{5}\left(-\cos u\Big|_1^3\right)\left(-\frac{1}{v}\Big|_4^{10}\right)=\frac{3}{100}(\cos 1-\cos 3) \\ &\approx 0.045.\end{aligned}$$