

Math 123 Abstract Mathematics I  
Homework 1

In the following problems assume only the validity of Peano axioms **P1-P7** as stated on pages 106 and 110. The hint of problem 1 can be used, after slight modification, also for the other problems.

1) Show that addition is associative.

Hint: For  $n \in \mathbb{N}$  let  $P(n)$  be the statement that  $\forall m, k \in \mathbb{N}, (m + k) + n = m + (k + n)$ . Then prove  $P(n)$  by induction for all  $n \in \mathbb{N}$ .

2) Show that for all  $m \in \mathbb{N}$ ,  $1 + m = m + 1$ .

3) Show that addition in  $\mathbb{N}$  is commutative.

4) Show that cancellation holds for addition in  $\mathbb{N}$ , i.e. for all  $k, m, n \in \mathbb{N}$ ,  $m + n = k + n \implies m = k$ .

**Solutions:**

1)