

Math 123 – Homework 3

Due date: 7 January 2009 Wednesday

Please take your homework solutions to room SA144, Ali Adalı's office before 17:00.

- Q-1)** For a finite group G , show that if $o(G)$ is even, then there is a non-trivial element $a \in G$ such that $a^{-1} = a$.
- Q-2)** Let $\phi : G \rightarrow H$ be a group homomorphism. Show that ϕ is one-to-one if and only if $\ker \phi = \{e\}$.
- Q-3)** Let $\phi : G \rightarrow H$ be a group homomorphism. Show that $\phi(G)$ is a subgroup of H and is isomorphic to the quotient group $G/\ker \phi$.
- Q-4)** Let $\theta \in S_n$ be a 2-cycle. Show that $\prod_{i < j} (x_i - x_j) = - \prod_{i < j} (x_{\theta(i)} - x_{\theta(j)})$.
- Q-5)** Let G be a finite group and H a subgroup with the property that $i(H)$ is the smallest prime p dividing the order of G . Show that H is a normal subgroup of G .
Hint: Show that G permutes the set of right cosets of H and that the kernel must be contained in H . Now use Lagrange's theorem together with the fact that no prime larger than or equal to p can divide $(p - 1)!$.

Grading: Each problem is 20 points.

Please forward any comments or questions to sertoz@bilkent.edu.tr
