Due Date: 26 October 2015, Monday
Time: Class time
Instructor: Ali Sinan Sertöz

NAME:..

STUDENT NO:.......................................

Math 202 Complex Analysis – Homework 2 – Solutions

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Please do not write anything inside the above boxes!

Check that there are **5** questions on your exam booklet. Write your name on top of every page. Show your work in reasonable detail. A correct answer without proper or too much reasoning may not get any credit. **Submit your solutions on this booklet only. Use extra pages if necessary.**

Rules for Homework and Take-Home Exams

(1) You may discuss the problems only with your classmates or with me. In particular you may not ask your assigned questions or any related question to online forums.

(2) You may use any written source be it printed or online. Google search is perfectly acceptable.

(3) It is absolutely mandatory that you write your answers alone. Any similarity with your written words and any other solution or any other source that I happen to know is a direct violation of honesty.

(4) You must obey the usual rues of attribution: all sources you use must be explicitly cited in such a manner that the source is easily retrieved with your citation. This includes any ideas you borrowed from your friends.

(5) Even if you find a solution online, you must rewrite it in your own narration, fill in the blanks if any, making sure that you exhibit your total understanding of the ideas involved.

Affidavit of compliance with the above rules: I affirm that I have complied with the above rules in preparing this submitted work.

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NAME: STUDENT NO:

Q-1) Prove or disprove Exercise 2.31 (8) on page 48: "Consider two antipodal points (x, y, u) and $(-x, -y, -u)$ on the Riemann sphere. Show that their stereographic projections z and z' are related by $zz' = -1$".

Solution:

This is wrong. What is correct is $z\bar{z}' = -1$. This can be seen either by directly writing down formulas or by a geometrical observation: The triangle in space formed by z, z' and N, the North pole, is a right triangle and has its hypothenuse in the plane passing through the origin. The line joining N to the origin is perpendicular to the hypothenuse. By Thales we know that $|z||z'| = 1$. So we know that if $z = re^{i\theta}$, then $z' = (1/r)e^{i(\theta+\pi)} = -(1/r)(1/e^{-i\theta})$, from which it follows that $z' = -1/\bar{z}$.

NAME: STUDENT NO:

Q-2) Fix four distinct points z_1, z_2, z_3, z_4 in $\mathbb{C}$. Let $\lambda = (z_1, z_2, z_3, z_4)$ be their cross-ratio. Let S_4 be the permutation group on $\{1, 2, 3, 4\}$, and define $\lambda_\sigma = (z_{\sigma(1)}, z_{\sigma(2)}, z_{\sigma(3)}, z_{\sigma(4)})$ be the corresponding cross-ratio for $\sigma \in S_4$. Find all λ_σ.

Solution:

A direct calculation will give that $\{\lambda_\sigma \mid \sigma \in S_4\} = \{\lambda, \frac{1}{\lambda}, 1-\lambda, \frac{1}{1-\lambda}, \frac{\lambda}{\lambda-1}, \frac{\lambda-1}{\lambda}\}$.

NAME: STUDENT NO:

Q-3) Find all zeros of $\sin z$ and $\cos z$ in the complex plane.

Solution:

Since $\sin z = \sin x \cosh y + i \cos x \sinh y = 0$ has only the solutions $x = n\pi$ and $y = 0$, for $n \in \mathbb{Z}$, the only zeros of the complex sine function are those of the real sine function.

On the other hand since $\cos z = \sin(z - \frac{\pi}{2})$, the only zeros of the complex cosine function are those of the real one.

NAME: STUDENT NO:

Q-4) For any $z \neq 0$ in the complex plane, calculate $(\exp \circ \log)(z)$ and $(\log \circ \exp)(z)$.

Solution:

We have $(\exp \circ \log)(z) = z$ and $(\log \circ \exp)(z) = z + i2n\pi$.

NAME: STUDENT NO:

Q-5) Calculate all values of i^{i^i} and $|i^{i^i}|$

Solution:

We have $i^i = \exp(i \log i) = \exp(i[i(\pi/2 + 2n\pi)])$, for $n \in \mathbb{Z}$. Hence $i^{i^i} = \exp(i^i \log i) = \exp(e^{-\pi/2-2n\pi}[i(\pi/2 + 2n\pi)])$. In the second $\log$ we choose the same branch as the previous $\log$, so we use the same integer n. It now follows from the expression for i^{i^i} that $|i^{i^i}| = 1$