MATH 202 Complex Analysis
Homework 1
Due date: 15 October 2021 Friday Class Time

Show your work in reasonable detail. It is important that you explain your solution in a convincing way. The grader can but will not do mind reading!

1) Calculate the following and give your answer in rectangular form $a + ib$ where a and b are real numbers.

(a) All cubic roots of i.

(b) $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{2021}$.

2) Let $f(z) = u(x, y) + iv(x, y)$ be an entire function and let c_1, c_2 be two real numbers. Assume that the curves $u(x, y) = c_1$ and $v(x, y) = c_2$ intersect at a point $z_0 = x_0 + iy_0$. Show that these two curves intersect at z_0 at right angles if $f'(z_0) \neq 0$.

3) Let $f(z)$ be analytic at z_0, and $g(z)$ be analytic at $w_0 = f(z_0)$. Show that $(g \circ f)$(z) is analytic at z_0 and moreover show that $(g \circ f)'(z_0) = g'(w_0)\, f'(z_0)$.

4) Consider the images of the hyperbolas $x^2 - y^2 = \pm c^2$ under the mapping $f(z) = z^2$, where $c > 0$. Show that you have two sheets on the image and show how these sheets are glued together so that f becomes one-to-one and onto this new surface.

5) Calculate the principal value of $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{i}$.